GATE-2015 (CIVIL ENGINEERING) Afternoon Session Paper with Solutions

- The Paper was held on 8th February, 2015 (Afternoon) and consists of 65 questions carrying 100 marks.
- General Aptitude section has 10 questions and carry a total of 15 marks. Q.1 – Q.5 carry 1 mark each, and questions Q.6 – Q.10 carry 2 marks each.
- Civil Engineering section has 55 questions and carry a total of 85 marks. Q.1 – Q.25 carry 1 mark each, and questions Q.26 – Q.55 carry 2 marks each.
- All efforts have been made to make this information as accurate as possible; Relevant/Authentic Text Books may be consulted for further information.
- Any discrepancy found may be brought to our notice by sending an e-mail to egyanbodh@gmail.com.
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Section: General Aptitude

1. Choose the most appropriate word from the options given below to complete the following sentence.

   The official answered _______ that the complaints of the citizen would be looked into.

   (A) respectably

   (B) respectfully

   (C) reputably

   (D) respectively

Ans (B) respectfully

2. Choose the statement where underlined word is used correctly.

   (A) The minister insured the victims that everything would be all right.

   (B) He ensured that the company will not have to bear any loss.

   (C) The actor got himself ensured against any accident.

   (D) The teacher insured students of good results.

Ans (B) He ensured that the company will not have to bear any loss.

- **Ensure** means to make sure or guarantee, whereas **insure** means to provide insurance (health, property etc.)

3. Which word is not a synonym for the word vernacular?

   (A) regional

   (B) indigenous

   (C) indigent

   (D) colloquial

Ans (C) indigent

- **Vernacular** is the language or dialect spoken by the ordinary people of a country or region. Synonyms are colloquial, indigenous and regional.
- **Indigent** means poor, needy etc.

4. Mr. Vivek walks 6 meters North-east, then turns and walks 6 meters South-east, both at 60 degrees to east. He further moves 2 meters South and 4 meters West. What is the straight distance in metres between the point he started from and the point he finally reached?

(A) $2\sqrt{2}$

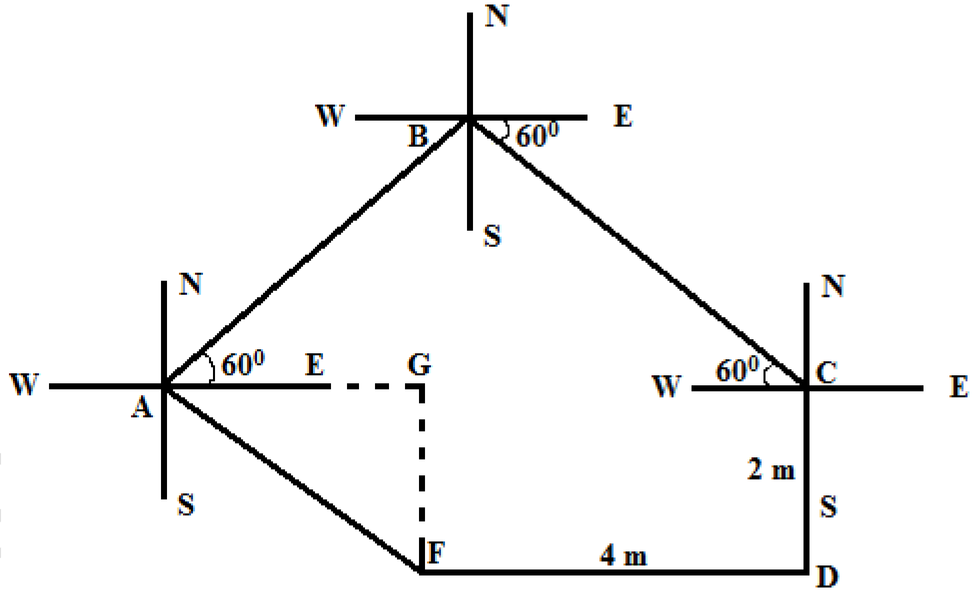
(B) 2

(C) $\sqrt{2}$

(D) $1/\sqrt{2}$

Ans (A) $2\sqrt{2}$

- The given conditions can be drawn as below (Vivek starts from A):



Triangle ABC is an equilateral triangle.

» AC = 6 m

From the diagram, GF = AG = 2 m

Therefore, AF = $\sqrt{2^2 + 2^2} = 2\sqrt{2}$ **m**

5. Four cards are randomly selected from a pack of 52 cards. If the first two cards are kings, what is the probability that the third card is a king?

   (A) 4 / 52

   (B) 2 / 50

   (C) (1 / 52) X (1 / 52)

   (D) (1 / 52) X (1 / 51) X (1 / 50)

Ans (B) 2 / 50

- Since the first two cards drawn are king, the remaining pack of 50 cards will have only 2 kings.

  Thus probability of third card to be king = $\frac{{}^{2}C_{1}}{{}^{50}C_{1}}$ = **2 / 50**

6. The word similar in meaning to 'dreary' is

   (A) cheerful

   (B) dreamy

   (C) hard

   (D) dismal

Ans (D) dismal

- **Dreary** means causing unhappiness or sad feelings,dull and poor. Hence answer is option (D) dismal.

7. The given question is followed by two statements; select the most appropriate option that solves the question.

   Capacity of a solution tank A is 70% of the capacity of tank B. How many gallons of solution are in tank A and tank B?

   Statements:

   (I) Tank A is 80% full and tank B is 40% full

   (II) Tank A if full contains 14,000 gallons of solution.

   (A) Statement I alone is sufficient

(B) Statement II alone is sufficient

(C) Either statement I or II alone is sufficient.

(D) Both the statements I and II together are sufficient.

Ans (D) Both the statements I and II together are sufficient.

- Let capacity of tank B = x

  So, capacity of tank A = 0.70 x

  Clearly from statement (II) only, total capacity (but not present quantity) of both tanks can be determined.

  0.7 x = 14000

  » x = 20000 gallons

  Now using statement (I), present quantity in both the tanks can be calculated.

  Hence, both the statements I and II together are sufficient.

8. How many four digit numbers can be formed with the 10 digits 0,1,2,….9 if no number can start with 0 and if repetitions are not allowed?

Ans 4536

- A four digit number will be of the form – ABCD (where A – Thousand's place, B – Hundred's place, C – Ten's place and D – One's place)

  Clearly A can not have 0 at its place.

  Thus possible number of options for A is 9 (1,2...9).

  Others can have any number including 0 and as no repetition is allowed

  Possible number of options for B = 9

  Possible number of options for C = 8

  Possible number of options for D = 7

  Hence, four digit numbers possible = 9 x 9 x 8 x 7 = **4536**

9. Read the following table giving sales data of five types of batteries for years 2006 to 2012:

Year	Type I	Type II	Type III	Type IV	Type V
2006	75	144	114	102	108
2007	90	126	102	84	126
2008	96	114	75	105	135
2009	105	90	150	90	75
2010	90	75	135	75	90
2011	105	60	165	45	120
2012	115	85	160	100	145

Out of the following, which type of battery achieved highest growth between the years 2006 and 2012?

(A) Type V

(B) Type III

(C) Type II

(D) Type I

Ans (D) Type I

- Growth for TYPE I battery = $\frac{115-75}{75}$ X 100 = **53.33%**

Growth for TYPE II battery = No growth, rather regression

Growth for TYPE III battery = $\frac{160-114}{114}$ X 100 = 40.35%

Growth for TYPE V battery = $\frac{145-108}{108}$ X 100 = 34.26%

Hence highest growth for TYPE I battery.

10. There are 16 teachers who can teach Thermodynamics (TD), 11 who can teach Electrical Sciences (ES), and 5 who can teach both TD and Engineering Mechanics (EM). There are a total of 40 teachers. 6 cannot teach any of the three subjects, i.e.

EM, ES or TD. 6 can teach only ES. 4 can teach all three subjects, i.e. EM, ES and TD. 4 can teach ES and TD. How many can teach both ES and EM but not TD?

(A) 1

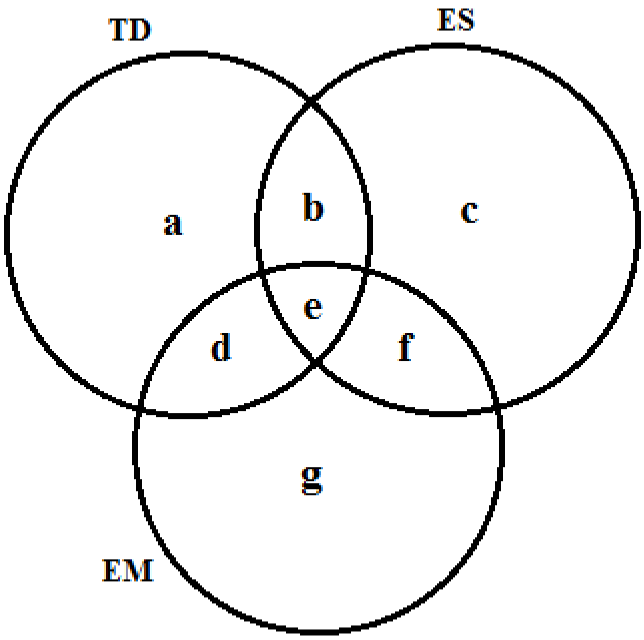
(B) 2

(C) 3

(D) 4

Ans (A) 1

- The given details can be arranged in a venn diagram as below:



From the diagram:

a + b + d + e = 16 ----------------------------------(A)

b + c + e + f = 11 ----------------------------------(B)

d + e = 5 ---(C)

a + b + c + d + e + f + g = 40 – 6 = 34 ----------(D)

c = 6; e = 4; b + e = 4

Clearly b = 0 as e = 4. On putting these values in equation (B), we get

f = 11 – 10 = 1

Hence, teachers who can teach both ES and EM but not TD = f = **1**

Section: Civil Engineering

1. While minimizing the function f(x) necessary and sufficient conditions for a point, x_0 to be a minima are:

   (A) $f'(x_0) > 0$ and $f''(x_0) = 0$

   (B) $f'(x_0) < 0$ and $f''(x_0) = 0$

   (C) $f'(x_0) = 0$ and $f''(x_0) < 0$

   (D) $f'(x_0) = 0$ and $f''(x_0) > 0$

Ans (D) $f'(x_0) = 0$ and $f''(x_0) > 0$

- Conditions for **minima**: $f'(x_0) = 0$ and $f''(x_0) > 0$

  Conditions for **maxima**: $f'(x_0) = 0$ and $f''(x_0) < 0$

2. In Newton-Raphson iterative method, the initial guess value (x_{ini}) is considered as zero while finding the roots of the equation: $f(x) = -2 + 6x - 4x^2 + 0.5x^3$. The correction, Δx, to be added to x_{ini} in the first iteration is________.

Ans 0.33

- According to Newton – Raphson method for f(X) = 0,

  $$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$

  $$» X_1 = X_{ini} - \frac{f(X_{ini})}{f'(X_{ini})}$$

  Here, $f(X) = f(x) = -2 + 6x - 4x^2 + 0.5x^3$

  » $f'(X) = 6 - 8x + 1.5x^2$

  Now, $X_{ini} = 0$, $f(X_{ini}) = f(0) = -2$ and $f'(X_{ini}) = f'(0) = 6$

  $$X_1 = 0 - \frac{-2}{6} = 0.33$$

  Hence correction, $\Delta x = X_1 - X_{ini} =$ **0.33**

3. Given, i = $\sqrt{-1}$, the value of the definite integral, I = $\int_0^{\pi/2} \frac{\cos x + i \sin x}{\cos x - i \sin x} dx$ is:

(A) 1

(B) -1

(C) i

(D) –i

Ans (C) i

- I = $\int_0^{\pi/2} \frac{\cos x + i \sin x}{\cos x - i \sin x} dx = \int_0^{\pi/2} \frac{e^{ix}}{e^{-ix}} dx$ (because $e^{ix} = \cos x + i \sin x$)

» I = $\int_0^{\pi/2} e^{2ix} dx = \left[\frac{e^{2ix}}{2i}\right]_0^{\pi/2} = \frac{e^{i\pi} - e^0}{2i}$ = (1/2i) (cosπ + i sinπ – 1)

» **I = -1/i = i** (because $i^2 = -1$)

4. $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^{2x}$ is equal to

(A) e^{-2}

(B) e

(C) 1

(D) e^2

Ans (D) e^2

- The given limit is, y = $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^{2x}$

Let x = 1/t

As x → ∞, t → 0

y = $\lim_{t\to 0}(1 + t)^{2/t}$

ln y = $\lim_{t\to 0}(2/t)$ ln (1 + t)

Since limit is of the form 0/0, we can apply L Hospital's rule

According to L Hospital's rule,

if f(a) = g(a) = 0, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

» $\ln y = \lim_{t\to 0}\frac{2}{1+t} = 2$

» $y = e^2$

Note the following formulas:

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \text{ and } \lim_{x\to 0}(1+x)^{1/x} = e$$

From above formula, $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x} = \left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right]^2 = e^2$

5. Let A = $[a_{ij}]$, $1 \le i, j \le n$ with $n \ge 3$ and $a_{ij} = i.j$. The rank of A is:
   (A) 0
   (B) 1
   (C) n – 1
   (D) n

Ans (B) 1

- The given matrix will be:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \\ 2 & 4 & 6 & 8 & 10 & \\ 3 & 6 & 9 & 12 & 15 & \ldots\ldots\ldots..j = n \\ 4 & 8 & 12 & 16 & 20 & \\ 5 & 10 & 15 & 20 & 25 & \end{bmatrix}$$

.... i = m

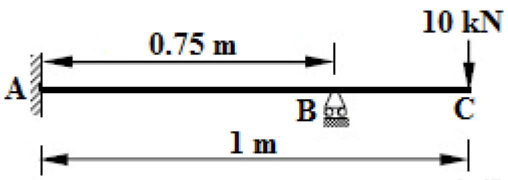
Applying $R_2 - 2R_1$, $R_3 - 3R_1$, $R_4 - 4R_1 \ldots R_m - mR_{m-1}$, all rows will become 0 except 1st.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \ldots\ldots\ldots\ldots j = n \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{bmatrix}$$

.... $i = m$

Hence Rank of matrix A = 1

6. A horizontal beam ABC is loaded as shown in the figure below. The distance of the point of contraflexure from end A (in m) is _________.



Ans 0.25 m

- $M_{BC} + 10 \times 0.25 = 0$

» $M_{BC} = -2.5$ kNm (anticlockwise)

Now at B, $M_{BA} + M_{BC} = 0$

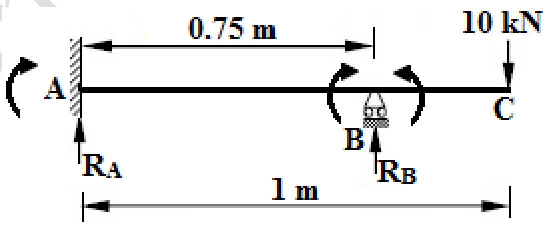
» $M_{BA} = -M_{BC} = 2.5$ kNm (clockwise)

$M_{AB} = (2EI/3)(2\theta_A + \theta_B) + M_{FAB}$

$M_{BA} = (2EI/3)(2\theta_B + \theta_A) + M_{FBA}$

In this case, $M_{FAB} = M_{FBA} = 0$; $\theta_A = 0$

» $M_{AB} = M_{BA}/2 = 2.50/2 = 1.25$ kNm (clockwise)



Taking moment about A,

$R_B \times 0.75 - 10 \times 1 - M_{AB} = 0$

» $R_B = 15$ kN (upward)

Also, $R_A + R_B = 10$

» R_A = - 5 kN (Downward)

Let the distance of point of contraflexure (M = 0) from A = x

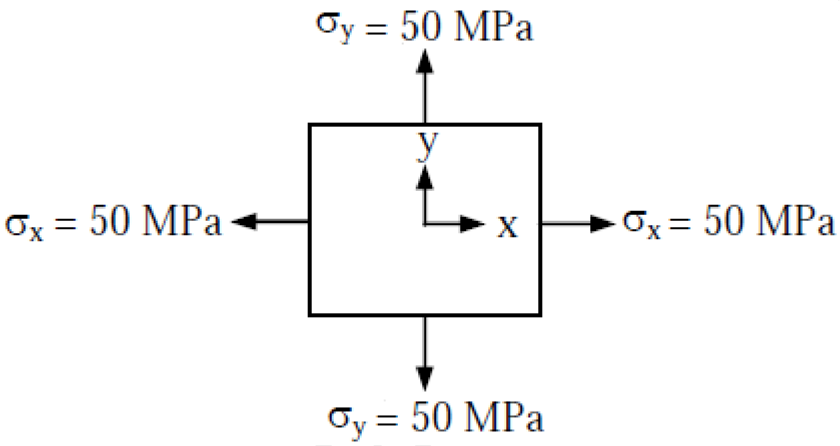
Taking moment from left hand side i.e. from A

$R_A (x) + M_{AB} = 0$

$- 5x + 1.25 = 0$

» x = 1.25 / 5 = 0.25 m

7. For the plane stress situation shown in the figure, the maximum shear stress and the plane on which it acts are:



(A) –50 MPa, on a plane 45^0 clockwise w.r.t. x-axis

(B) –50 MPa, on a plane 45^0 anti-clockwise w.r.t. x-axis

(C) 50 MPa, at all orientations

(D) Zero, at all orientations

Ans (D) Zero, at all orientations

- Maximum shear stress, $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Here, σ_x = 50 MPa, σ_y = 50 MPa

» $\tau_{max} = \sqrt{\left(\frac{50 - 50}{2}\right)^2 + 0} = 0$

Note: The formulas for principal stresses and maximum shear stress:

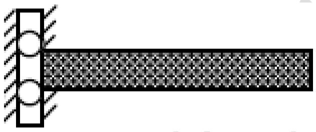
1) Principal stresses, $\boldsymbol{\sigma} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Position of principal planes, $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

2) Maximum shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Position of maximum shear stress plane, $\theta_s = 45^0 + \theta_{p1}$

8. A guided support as shown in the figure below is represented by three springs (horizontal, vertical and rotational) with stiffness k_x, k_y, and k_θ respectively. The limiting values of k_x, k_y, and k_θ are :



(A) ∞, 0, ∞

(B) ∞, ∞, ∞

(C) 0, ∞, ∞

(D) ∞, ∞, 0

Ans (A) ∞, 0, ∞

- As there is a guided support, the force or restrain in vertical (y) direction is 0. In the other two given directions, the restrain can go upto infinity.

  Since, stiffness (k) = F/x

  Hence, the limiting values will be $k_x = \infty$; $k_y = 0$ and $k_z = \infty$

9. A column of size 450 mm × 600 mm has unsupported length of 3.0 m and is braced against side sway in both directions. According to IS 456: 2000, the minimum eccentricities (in mm) with respect to major and minor principal axes are :

(A) 20.0 and 20.0

(B) 26.0 and 21.0

(C) 26.0 and 20.0

(D) 21.0 and 15.0

Ans (B) 26.0 and 21.0

- As per IS 456:2000, clause 25.4, Minimum eccentricity of columns is given by

e = (Unsupported length of column/500 + Lateral dimension/30) $\geq$ 20 mm

$e = \frac{L}{500} + \frac{b \text{ or } d}{30}$, subjected to $e \geq 20$ mm

Thus, $e_{max} = \frac{3000}{500} + \frac{600}{30} = \mathbf{26}$

$e_{min} = \frac{3000}{500} + \frac{450}{30} = \mathbf{21}$

10. Prying forces are :

(A) shearing forces on the bolts because of the joints

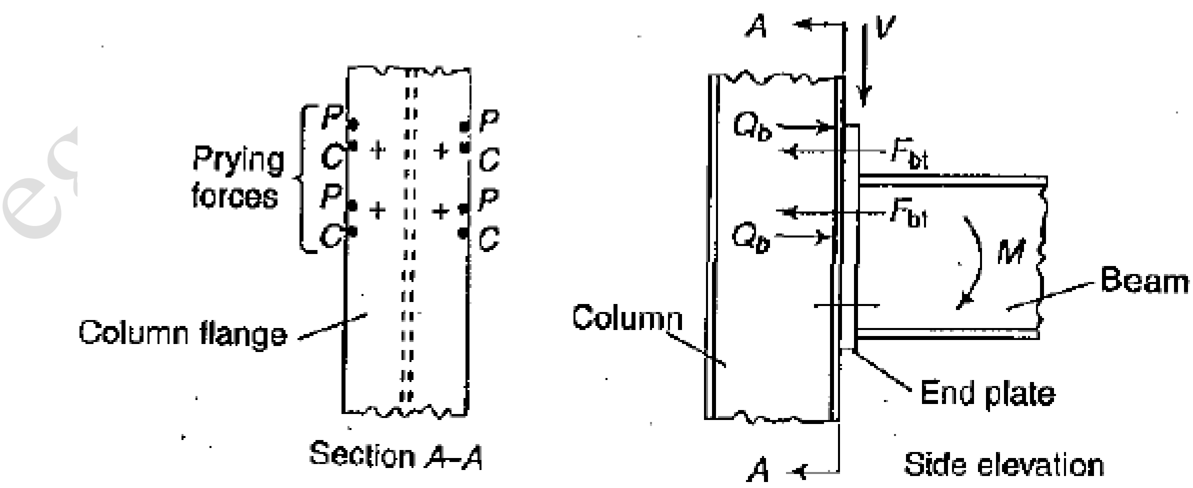
(B) tensile forces due to the flexibility of connected parts

(C) bending forces on the bolts because of the joints

(D) forces due to friction between connected parts

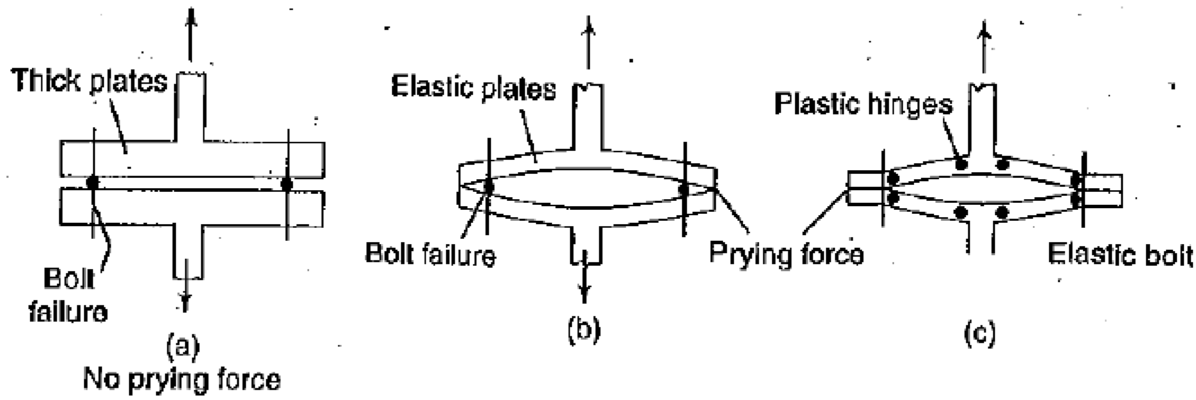
Ans (B) tensile forces due to the flexibility of connected parts

- Moment resisting beam-to-column connections often contain regions in which the bolts will be required to transfer load by direct tension, such as the upper bolts in the end plate connection as shown in Fig. below.



In the design of such connections, we should consider an additional force induced in the bolts as a result of so-called 'prying action' (Douty & McGuire 1965, Agerskov 1979, Holmes & Mattin 1983, Subramanian 1984). These additional prying forces induced in the bolts are mainly due to the flexibility of connected plates.

Thus, in a simple T-stub connection as shown in Fig. below,



Failure modes due to prying forces

the prying force will develop only when the ends of the flanges are in contact due to the external load, as shown in Figs (b) and (c). The plastic hinges do not always form before bolt failure.

11. A steel member ‘M’ has reversal of stress due to live loads, whereas another member ‘N’ has reversal of stress due to wind load. As per IS 800:2007, the maximum slenderness ratio permitted is:

(A) less for member ‘M’ than that of member ‘N’

(B) more for member ‘M’ than for member ‘N’

(C) same for both the members

(D) not specified in the Code

Ans (A) less for member ‘M’ than that of member ‘N’

- As per IS 800:2007, clause 3.8, Table-3

The values of M and N are 180 and 350 respectively.

Member	Max. Effective slenderness ratio
A tension member in which a reversal of direct stress occurs due to loads other than wind or seismic forces	180
A member normally acting as a tie in a roof truss or a bracing system not considered. effective when subject to possible reversal of stress into compression resulling from the action of wind or earthquake forces	350

Hence, option (A) is correct.

12. If the water content of a fully saturated soil mass is 100%, the void ratio of the sample is:

(A) less than specific gravity of soil

(B) equal to specific gravity of soil

(C) greater than specific gravity of soil

(D) independent of specific gravity of soil

Ans (B) equal to specific gravity of soil

- We know that eS = wG

  Here, S = 100%, w = 100%

  » e = G

13. In friction circle method of slope stability analysis, if r defines the radius of the slip circle, the radius of friction circle is:

(A) r sinϕ

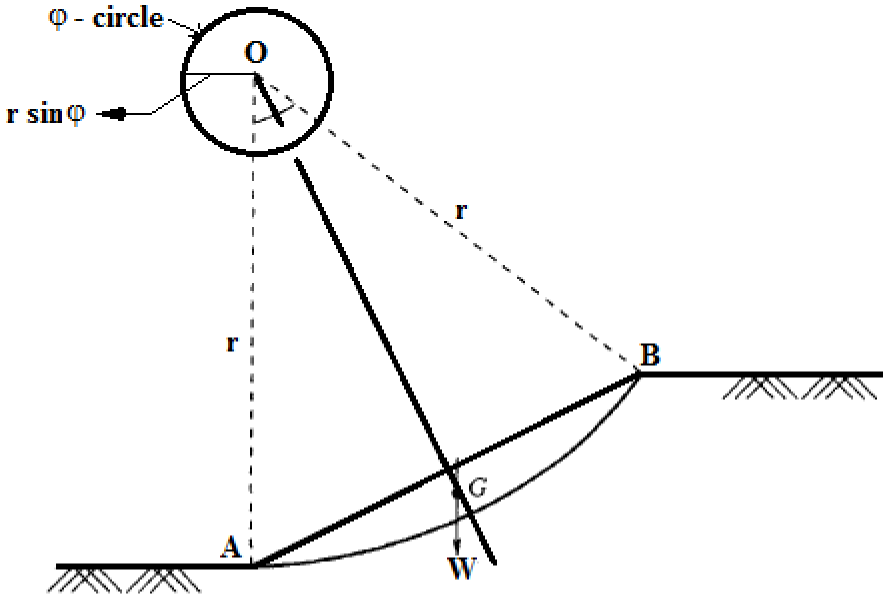
(B) r

(C) r cosϕ

(D) r tanϕ

Ans (A) r sinϕ

- The friction circle method is useful for the stability analysis of slopes made of homogeneous soils. In this method, the slip surface is assumed to be an arc of a circle.



14. Net ultimate bearing capacity of a footing embedded in a clay stratum

(A) increases with depth of footing only

(B) increases with size of footing only

(C) increases with depth and size of footing

(D) is independent of depth and size of footing

Ans (D) is independent of depth and size of footing

- As per Terzaghi's Bearing Capacity Theory,

  Ultimate Bearing Capacity, $q_u = c\,N_c + q\,N_q + 0.5\,\gamma\,B\,N_\gamma$

  For clay, $\phi = 0$

  Thus, $N_c = 5.7$, $N_q = 1$ and $N_\gamma = 0$

  » $q_u = 5.7\,c + q = 5.7\,c + \gamma\,D_f$

  Hence, Net ultimate bearing capacity

  $q_{nu} = q_u - \gamma\,D_f =$ **5.7 c (independent of depth and size of footing)**

15. Surcharge loading required to be placed on the horizontal backfill of a smooth retaining vertical wall so as to completely eliminate tensile crack is:

(A) 2c

(B) $2ck_a$

(C) $2c\sqrt{k_a}$

(D) $2c / \sqrt{k_a}$

Ans (D) $2c / \sqrt{k_a}$

- The active earth pressure is given by,

  $P_a = K_a\gamma H - 2c\sqrt{K_a}$

  Let Z_c is the depth of tensile crack, then

  $K_a\gamma Z_c - 2c\sqrt{K_a} = 0$

  » $Z_c = 2c / \gamma\sqrt{K_a}$

  Hence, surcharge loading to eliminate tensile crack = γZ_c = $\mathbf{2c / \sqrt{K_a}}$

16. The relationship between the length scale ratio (L_r) and the velocity scale ratio (V_r) in hydraulic models, in which Froude dynamic similarity is maintained, is:

(A) $V_r = L_r$

(B) $L_r = \sqrt{V_r}$

(C) $V_r = (L_r)^{1.5}$

(D) $V_r = \sqrt{L_r}$

Ans (D) $V_r = \sqrt{L_r}$

- $F = \dfrac{V_m}{\sqrt{gL_m}} = \dfrac{V_p}{\sqrt{gL_p}}$

  » $\dfrac{V_m}{V_p} = \dfrac{\sqrt{L_m}}{\sqrt{L_p}}$

  » $\mathbf{V_r = \sqrt{L_r}}$

17. A nozzle is so shaped that the average flow velocity changes linearly from 1.5 m/s at the beginning to 15 m/s at its end in a distance of 0.375 m. The magnitude of the convective acceleration (in m/s^2) at the end of the nozzle is _________.

Ans 540

- Let V(u,v,w) be the velocity of a particle at (x,y,z)

  So, Convective acceleration (a_c) = $a_x i + a_y j + a_z k$

  Where, $a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$

  $a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$

  $a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$

  Here, v = w = 0; so $a_y = a_z = 0$

  $a_c = a_x = u\frac{\partial u}{\partial x} = 15 \text{ x } \frac{15-1.5}{0.375}$ = **540 m/s²** (because at the end of nozzle, u = 15 m/s)

18. A hydraulic jump takes place in a frictionless rectangular channel. The pre-jump depth is y_p. The alternate and sequent depths corresponding to y_p are y_a and y_s respectively. The correct relationship among y_p, y_a and y_s is:

    (A) $y_a < y_s < y_p$

    (B) $y_p < y_s < y_a$

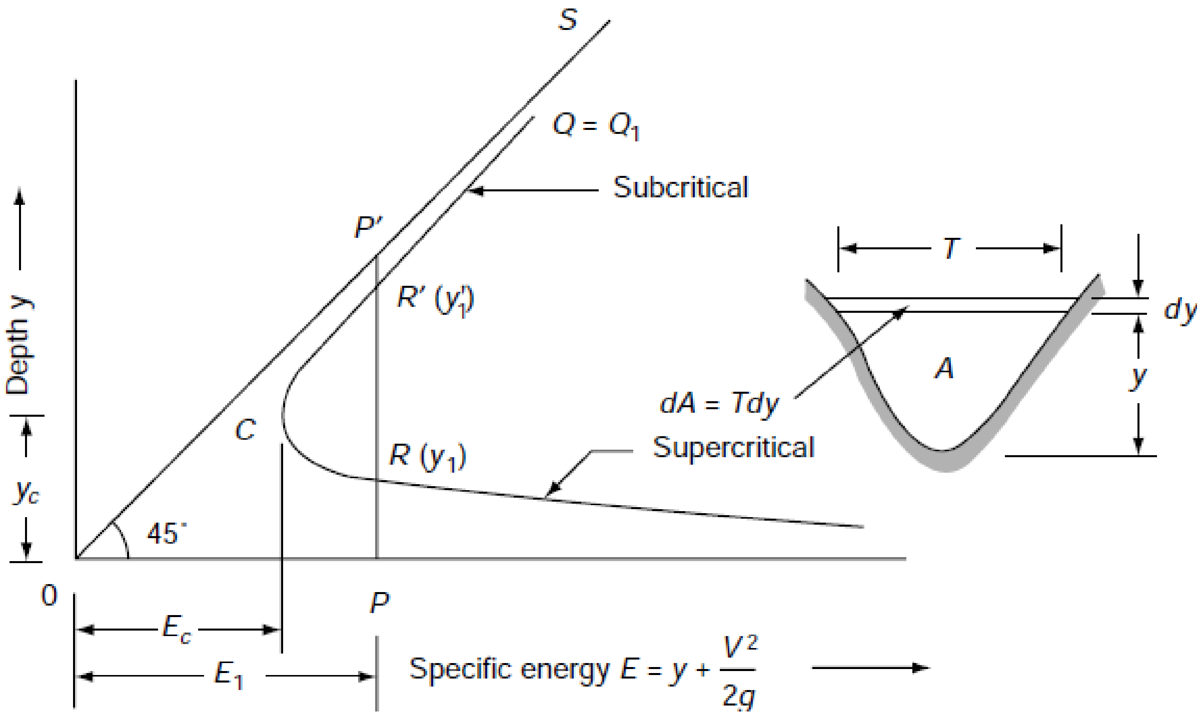
    (C) $y_p < y_s = y_a$

    (D) $y_p = y_s = y_a$

Ans (B) $y_p < y_s < y_a$

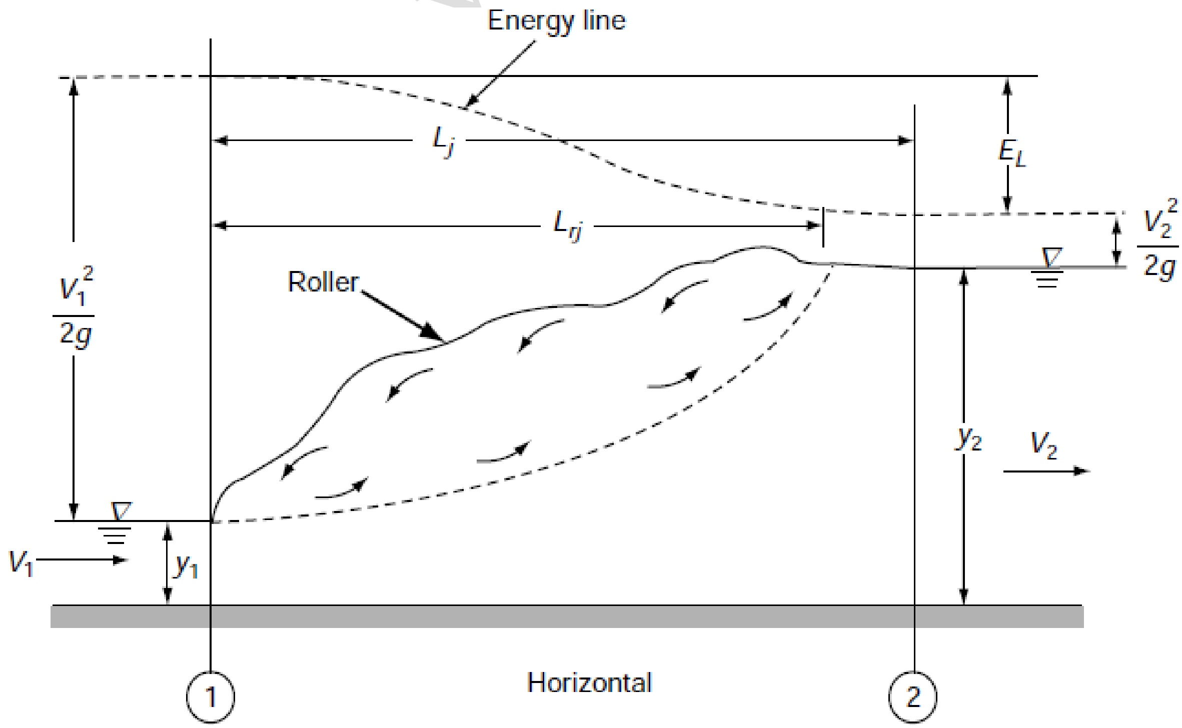
- Specific energy is given by, $E = y + V^2/2g = y + Q^2/2gA$

  Since it is a quadratic equation, it will give two values of y. These two possible depths having the same specific energy are known as **alternate depths**.



Here, y_1 and y_1' represents the alternate depths.

On the other hand, the two depths at the ends of a hydraulic jump are called **sequent depths**.



Here, y_1 and y_2 are sequent depths.

A hydraulic jump occurs when a supercritical stream meets a subcritical stream of sufficient depth. The supercritical stream jumps up to meet its alternate depth. While doing so it generates considerable disturbances in the form of large-scale eddies and a reverse flow roller with the result that the jump falls short of its alternate depth.

In other words due to energy losses, sequent depth is less than alternate depth.

Hence, **option (B)** $\mathbf{y_p < y_s < y_a}$ is correct.

19. The relationship between porosity (η), specific yield (S_y) and specific retention (S_r) of an unconfined aquifer is:

(A) $S_y + S_r = \eta$

(B) $S_y + \eta = S_r$

(C) $S_r + \eta = S_y$

(D) $S_y + S_r + \eta = 1$

Ans (A) $S_y + S_r = \eta$

- **Specific yield (S_y)** $= \dfrac{\text{Volume of water drained by gravity}}{\text{Total volume of the aquifer}} = \dfrac{V_{WY}}{V}$

  Specific Retention (S_r) $= \dfrac{\text{Volume of water retained}}{\text{Total volume of the aquifer}} = \dfrac{V_{WR}}{V}$

  Porosity (η) $= \dfrac{\text{Total Volume of water}}{\text{Total volume of the aquifer}} = \dfrac{V_W}{V}$

  Now, $V_W = V_{WY} + V_{WR}$

  » $\boldsymbol{\eta = \dfrac{V_W}{V} = \dfrac{V_{WY} + V_{WR}}{V} = S_y + S_r}$

20. A groundwater sample was found to contain 500 mg/L total dissolved solids (TDS). TDS (in %) present in the sample is __________.

Ans 0.05

- Density of water = 1 g/cc = 10^6 mg/L

  Therefore, TDS(%) $= \dfrac{500}{10^6} \times 100 =$ **0.05%**

21. SO_2 and CO adversely affect

    (A) oxygen carrying capacity of blood and functioning of lungs respectively

    (B) functioning of the respiratory system and brain respectively

    (C) functioning of the respiratory system and oxygen carrying capacity of blood respectively

    (D) functioning of air passages and chest respectively

Ans (C) functioning of the respiratory system and oxygen carrying capacity of blood respectively.

22. A superspeedway in New Delhi has among the highest super-elevation rates of any track on the Indians Grand Prix circuit. The track requires drivers to negotiate turns with a radius of 335 m and 33^0 banking. Given this information, the coefficient of side friction required in order to allow a vehicle to travel at 320 km/h along the curve is:

    (A) 1.761

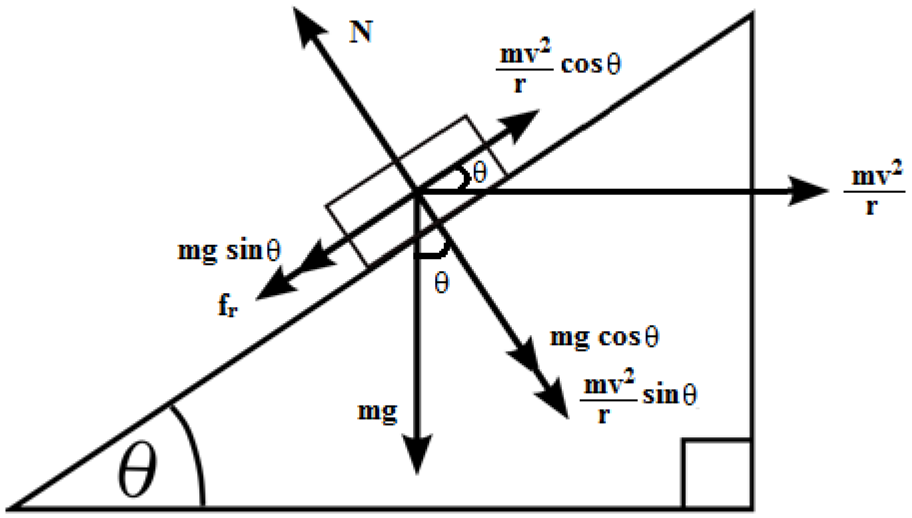
    (B) 0.176

    (C) 0.253

    (D) 2.530

Ans (A) 1.761

- The free body diagram of the vehicle is shown below:



Where, f_r = frictional force = fN (f is the coefficient of friction)

v = velocity of vehicle = 320 km/h = 320 x 1000 / 3600 = 88.89 m/s

$$N = mg\cos\theta + \frac{mv^2}{r}\sin\theta$$

$$mg\sin\theta + f_r = \frac{mv^2}{r}\cos\theta$$

$$mg\sin\theta + fN = \frac{mv^2}{r}\cos\theta$$

$$mg\sin\theta + f\left(mg\cos\theta + \frac{mv^2}{r}\sin\theta\right) = \frac{mv^2}{r}\cos\theta$$

$$» \frac{v^2}{rg} = \frac{f + \tan\theta}{1 - f\tan\theta}$$

Here, $\theta = 33^0$, v = 88.89 m/s, r = 335 m and g = 9.81 m/s^2

$$» \frac{88.89^2}{335 \times 9.8} = \frac{f + \tan 33^0}{1 - f\tan 33^0}$$

» f = 0.686

This is the minimum value of coefficient of friction required. Any value above it will be ok. In the given question, the closest value is 1.761. Hence option (A) is correct.

23. The following statements are made related to the lengths of turning lanes at signalised intersections:

(i) 1.5 times the average number of vehicles (by vehicle type) that would store in turning lane per cycle during the peak hour

(ii) 2 times the average number of vehicles (by vehicle type) that would store in turning lane per cycle during the peak hour

(iii) Average number of vehicles (by vehicle type) that would store in the adjacent through lane per cycle during the peak hour

(iv) Average number of vehicles (by vehicle type) that would store in all lanes per cycle during the peak hour

As per the IRC recommendations, the correct choice for design length of storage lanes is:

(A) Maximum of (ii and iii)

(B) Maximum of (i and iii)

(C) Average of (i and iii)

(D) Only (iv)

Ans (B) Maximum of (i and iii)

- As per IRC SP 041-1994, clause 4.8.1

  Storage lanes are generally more important in urban areas where volume of right turning traffic is high and if not catered for, blocks the through traffic. Normal design procedure provides for storage length based on 1 .5 times the average number of vehicles (by vehicle type) that would store in turning lane at peak hour. At the same time the concurrent through lane storage must also be kept in view, as it may happen that entry to turning lane may become inaccessibile due to queued vehicles in through lane. **Hence option (B) is correct.**

  One example is given in the code after the clause 4.8.1, which can be studied for better understanding. All IS and IRC codes can be downloaded from here: **https://law.resource.org/pub/in/manifest.in.html**

24. In a leveling work, sum of the Back Sight (B.S.) and Fore Sight (F.S.) have been found to be 3.085 m and 5.645 m respectively. If the Reduced Level (R.L.) of the starting station is 100.000 m, the R.L. (in m) of the last station is _________.

Ans 97.44

- $\sum$ FS - $\sum$ BS = RL of starting station – RL of last station

  5.645 – 3.085 = 100 – RL of last station

  » RL of last station = 97.44 m

25. The combined correction due to curvature and refraction (in m) for a distance of 1 km on the surface of Earth is

    (A) 0.0673

    (B) 0.673

    (C) 7.63

    (D) 0.763

Ans (A) 0.0673

- Combined correction due to curvature and refraction (in m)

  $C = 0.0673\ d^2$ (Where d is in km)

  » $C = 0.0673 \times 1^2 = 0.0673$ m

26. The probability density function of a random variable, X is

$f(x) = \frac{x}{4}(4 - x^2)$ for $0 \le x \le 2$

$= 0$ otherwise

The mean, μ_x of the random variable is __________.

Ans 1.067

- In case of continuous distribution, Mean of x is given by

  Mean (x) = $\mu_x = \int_{-\infty}^{\infty} x\, f(x)dx$

  In this case, $\mu_x = \int_0^2 x\, f(x)dx = \int_0^2 \frac{x^2}{4}(4 - x^2)\, dx$ (because for other values f(x) = 0)

  » $\mu_x = \left[\frac{x^3}{3} - \frac{x^5}{20}\right]_0^2 = 16/15 =$ **1.067**

27. Consider the following second order linear differential equation

$\frac{d^2y}{dx^2} = -12x^2 + 24x - 20$

The boundary conditions are : at x = 0, y = 5 and at x = 2, y = 21

The value of y at x = 1 is ___________.

Ans 18

- $\frac{d^2y}{dx^2} = -12x^2 + 24x - 20$

  Integrating both sides w.r.t x, we get

  $\frac{dy}{dx} = -4x^3 + 12x^2 - 20x + C_1$

Again Integrating both sides w.r.t x, we get

$y = - x^4 + 4x^3 - 10x^2 + C_1x + C_2$ --------------------------(A)

Now, at x = 0, y = 5; putting these values in equation (A), we get

$C_2 = 5$

Further, at x = 2, y = 21; putting these values in equation (A), we get

$21 = - 2^4 + 4 (2)^3 - 10 (2)^2 + C_1 (2) + 5$

» $C_1 = 20$

Thus, equation (A) becomes, $y = - x^4 + 4x^3 - 10x^2 + 20x + 5$

So, at x = 1

$\mathbf{y = - 1^4 + 4 (1)^3 - 10 (1)^2 + 20 (1) + 5 = 18}$

28. The two Eigen Values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & p \end{bmatrix}$ have a ratio of 3:1 for p = 2. What is another value of p for which the Eigen values have the same ratio of 3:1?

(A) -2

(B) 1

(C) 7 / 3

(D) 14 / 3

Ans (D) 14 / 3

- Let the eigen values are 3k and k

Sum of eigen values = sum of elements on main diagonal

» $4k = 2 + p$ -----------------------------(A)

Product of eigen values = Determinant of matrix

» $3k^2 = 2p - 1$ ---------------------------(B)

From equations (A) and (B)

$16 / 3 = (2 + p)^2 / (2p - 1)$

» $3p^2 - 20p + 28 = 0$

$(p - 2) (3p - 14) = 0$

Hence values of p are 2 and 14/3.

29. For step-size, $\Delta x = 0.4$, the value of following integral using Simpson's 1/3 rule is ____________.

$$\int_0^{0.8} (0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5)\, dx$$

Ans 1.367

- According to Simpson's 1/3 rule,

$$\int_{x_0}^{x_n} f(x)\, dx = \int_{x_0}^{x_0 + nh} f(x)\, dx$$

$$= \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + .. + y_{n-2})]$$

Where h = step size = $(x_n - x_0) / n$ and n is even

In the given question, $h = \Delta x = 0.4$; $n = 2$; $x_0 = 0$, $x_1 = 0.4$, $x_2 = 0.8$;

Given, $I = \int_0^{0.8} (0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5)\, dx$

$$= \frac{h}{3}(y_0 + 4y_1 + y_2)$$

$y_0 = 0.2$;

$y_1 = 0.2 + 25 (0.4) - 200 (0.4)^2 + 675 (0.4)^3 - 900 (0.4)^4 + 400 (0.4)^5 = 2.456$

$y_2 = 0.2 + 25 (0.8) - 200 (0.8)^2 + 675 (0.8)^3 - 900 (0.8)^4 + 400 (0.8)^5 = 0.232$

Hence, $\mathbf{I = \frac{0.4}{3} [0.2 + 4 (2.456) + 0.232] = 1.367}$

30. In a system, two connected rigid bars AC and BC are of identical length, L with pin supports at A and B. The bars are interconnected at C by a frictionless hinge. The rotation of the hinge is restrained by a rotational spring of stiffness, k. The system initially assumes a straight line configuration, ACB. Assuming both the bars as weightless, the rotation at supports, A and B, due to a transverse load, P applied at C is:

(A) PL/4k

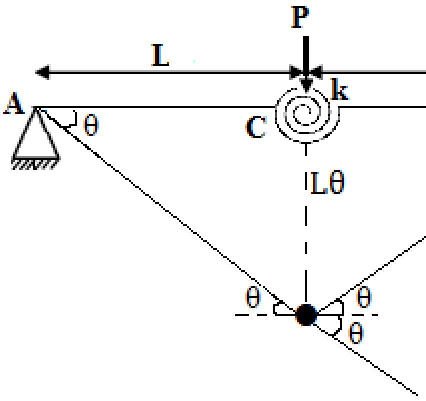
(B) PL/2k

(C) P/4k

(D) Pk/4L

Ans (A) PL/4k

- The given conditions can be represented as below:



When the rotation of each beam is δθ, the distance moved by point C = Lδθ

» Work done by the force P = PLδθ

Strain energy stored in the spring = $\delta[(1/2)\ k\ (\text{rotation})^2] = \delta[(1/2)\ k\ (2\theta)^2]$

For equilibrium, $\delta[(1/2)\ k\ (2\theta)^2] = PL\delta\theta$

» $2k\delta(\theta^2) = PL\delta\theta$

$4k\theta\delta\theta = PL\delta\theta$

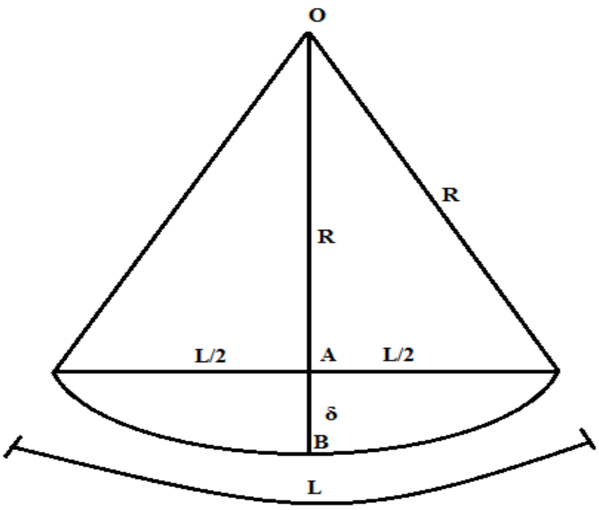
$4k\theta = PL$

» **$\theta = PL/4k$**

31. A simply supported reinforced concrete beam of length 10 m sags while undergoing shrinkage. Assuming a uniform curvature of 0.004 m^{-1} along the span, the maximum deflection (in m) of the beam at mid-span is ____________.

Ans 0.05

- Representing the beam as a single line:



Length of the beam = L

Deflection of the beam at mid point = AB = δ

Radius of curvature = OB = R

From pythagorus theorem,

$R^2 = (L/2)^2 + (R - \delta)^2$

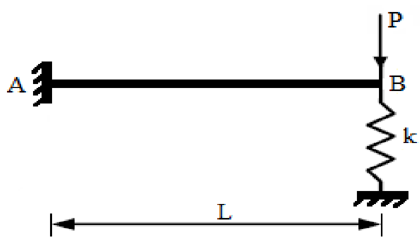
$2R\delta - \delta^2 = L^2/4$ (Neglect δ^2, as it is very small)

$\delta = L^2/8R$

Here, L = 10 m and 1/R = 0.004 m^{-1}

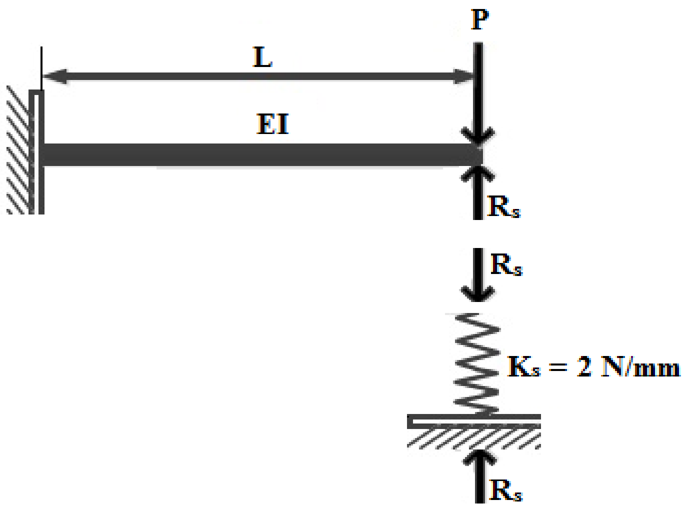
» $\delta = 10^2 \times 0.004 / 8 = 0.05$ m

32. A steel strip of length, L = 200 mm is fixed at end A and rests at B on a vertical spring of stiffness, k = 2 N/mm. The steel strip is 5 mm wide and 10 mm thick. A vertical load, P = 50 N is applied at B, as shown in the figure. Considering E = 200 GPa, the force (in N) developed in the spring is____________.

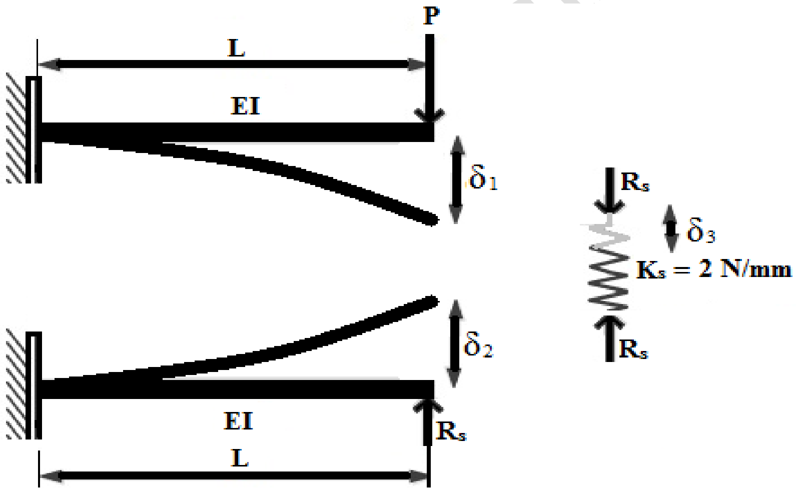


Ans 3

- The free body diagram is shown below: R_s is the reaction in spring.



The total deflections due to load are shown in the diagram given below:



δ_1 = Downward deflection due to load P (causing compression in spring) = $PL^3/3EI$

δ_2 = Upward deflection due to reaction R_s (causing elongation in spring) = $R_sL^3/3EI$

δ_3 = Shortening in spring due to reaction $R_s = R_s/K_s = 2R_sL^3/3EI$

From Compatibility Equation,

$\delta_3 = \delta_1 - \delta_2$

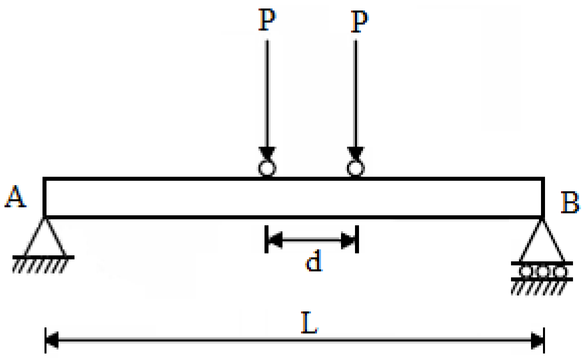
$R_s / K_s = PL^3/3EI - R_sL^3/3EI$

Here K_s = 2 N/mm, P = 50 N, L = 200 mm, E = 2 x 10^5 N/mm^2, I = 5 x 10^3/ 12 mm^4

On putting values in the above equation, we get

» R_s = 3 N

33. A simply supported beam AB of span, L = 24 m is subjected to two wheel loads acting at a distance, d = 5 m apart as shown in the figure below. Each wheel transmits is a load, P = 3 kN and may occupy any position along the beam . If the beam is an I-section having section modulus, S = 16.2 cm^3, the maximum bending stress (in GPa) due to the wheel loads is___________.



Ans 1.783

- From the flexural formula,

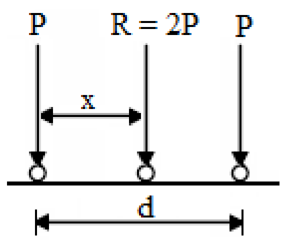
$$\frac{\sigma}{y} = \frac{M}{I}$$

$$» \sigma = \frac{M}{S}$$

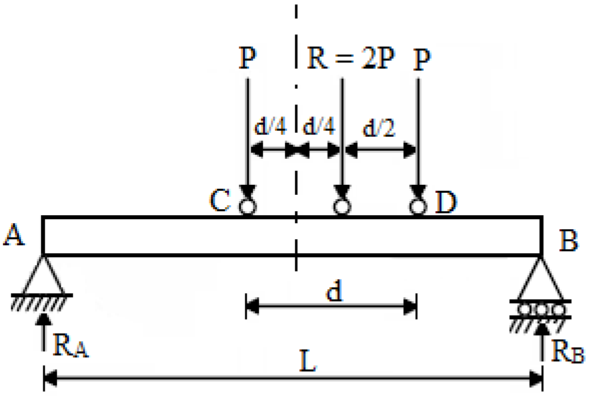
The maximum bending stress will be at the point of maximum bending moment.

The maximum bending will occur under one of the wheel loads. Further, the maximum bending moment will take place when the resultant of the forces and load under consideration (load P) is equidistant from the centre of the beam. The resultant of the two loads will pass through the mid point between them.

x = [P (0) + P (d)] / 2P = d/2



Hence, the resultant and load P should be at a distance of d/4 from the centre of the beam. While determining the reactions, **take either the loads or resultant force in calculation.** The situation is as shown below:



R_A (L) = P (L/2 + d/4) + P (L/2 – 3d/4) = P (L – d/2)

R_A = 2.6875 kN (because L = 24 m, P = 3 kN, d = 5 m)

» R_B = 6 – 2.6875 = 3.3125 kN

Moment under load at C = 2.6875 x 10.75 = 28.891 kNm

Moment under load at D = 3.3125 x 8.25 = 27.328 kNm

Hence, maximum bending stress = $\sigma = \frac{M}{S} = \frac{28.891 \times 10^6}{16.2 \times 10^3}$ MPa = 1.783×10^3 MPa

» **σ = 1.783 GPa**

34. According to the concept of Limit State Design as per IS 456:2000, the probability of failure of a structure is __________.

Ans 0.0975

- As per IS 456:2000, Clause 36

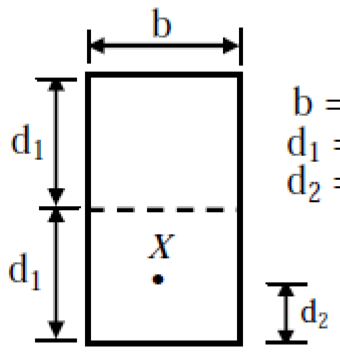
36.1 **Characteristic Strength of Materials** - The term 'characteristic strength' means that value of the strength of the material below which not more than 5 percent of the test results are expected to fall. Thus Probability of survival, P(A) = 0.95 and Probability of failure, $P(\bar{A})$ = 1 – 0.95 = 0.05

36.2 **Characteristic Loads** - The term 'characteristic load' means that value of load which has a 95 percent probability of not being exceeded during the life of the structure. Thus Probability of survival, P(B) = 0.95 and Probability of failure, $P(\bar{B})$ = 1 – 0.95 = 0.05

Hence probability of failure of a structure = $P(\bar{A}) \times P(B) + P(A) \times P(\bar{B}) + P(\bar{A}) \times P(\bar{B})$

» P = 0.05 x 0.95 + 0.95 x 0.05 + 0.05 x 0.05 = 0.0975

35. In a pre-stressed concrete beam section shown in the figure, the net loss is 10% and the final pre-stressing force applied at X is 750 kN. The initial fiber stresses (in N/mm^2) at the top and bottom of the beam were:



(A) 4.166 and 20.833

(B) -4.166 and -20.833

(C) 4.166 and -20.833

(D) -4.166 and 20.833

Ans (D) -4.166 and 20.833

- Area of cross-section, A = 250 x 400 = 100000 mm^2

Final Prestress force, P’ = 750 kN = 750000 N

Due to 10% losses, Initial Prestress force, P = P’/0.9 = 833333.33 N

Eccentricity, e = 100 mm

Moment of inertia, I = 250 x 400^3 / 12 = 1.333 x 10^9 mm^4

Section modulus (Z_b and Z_t) = (1.333 x 10$^{9)}$ / 200 = 6.665 x 10^6 mm^3

Direct stress due to Prestress = P/A = 833333.33 / 100000 = 8.333 N/mm^2

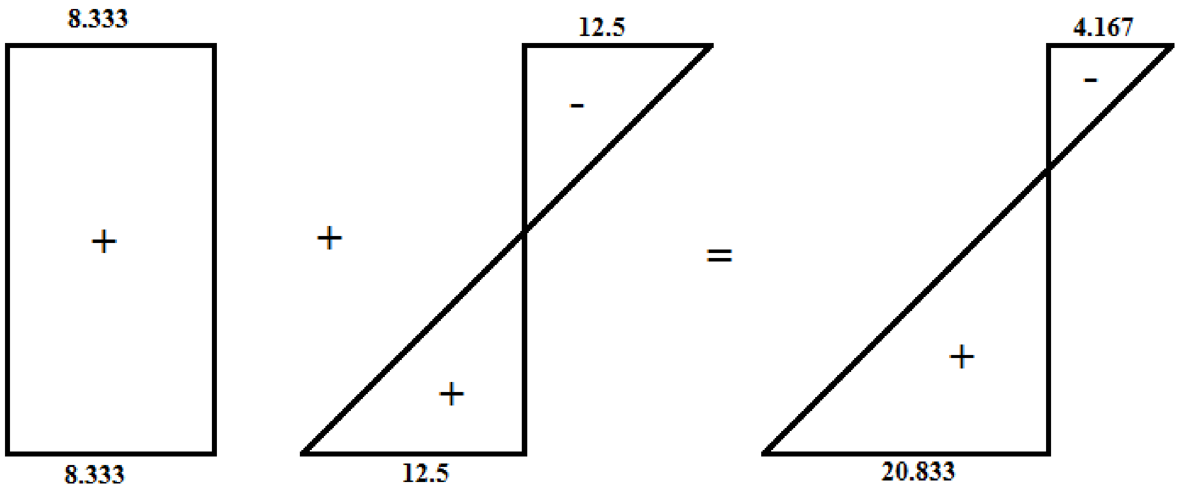
Bending stress due to Prestress = Pe/Z = (833333.33 x 100) / (6.665 x 10^6)

= 12.5 N/mm^2

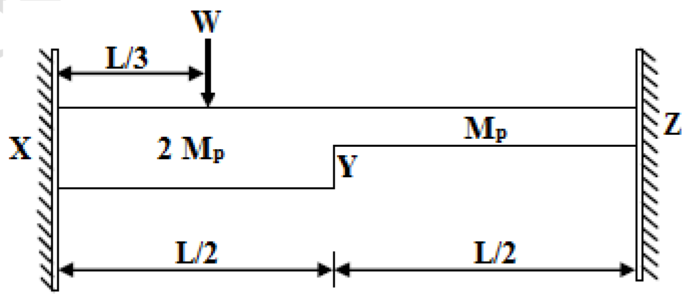
Resultant stress at mid span (top), σ_t = 8.333 – 12.5 = - 4.167 N/mm^2 (tension)

Resultant stress at mid span (bottom), σ_b = 8.333 + 12.5

= 20.833 N/mm^2 (compression)



36. A fixed end beam is subjected to a load, W at 1/ 3rd span from the left support as shown in the figure. The collapse load of the beam is



(A) 16.5 M_P/L

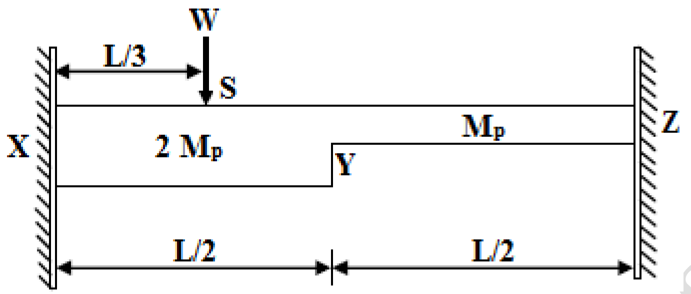
(B) 15.5 M_P/L

(C) 15.0 M_P/L

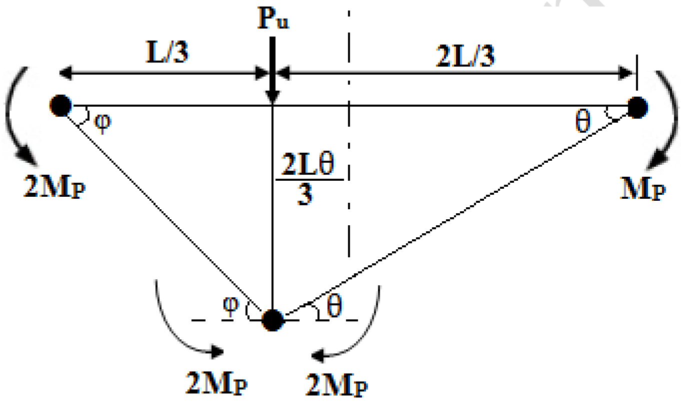
(D) 16.0 M_P/L

Ans (C) 15.0 M_P/L

- The given beam is:



There are two possible collapse mechanisms. In the first mechanism, hinges may form at X, S and Z.



From the figure above,

$2L\theta/3 = L\varphi/3$

» $\varphi = 2\theta$

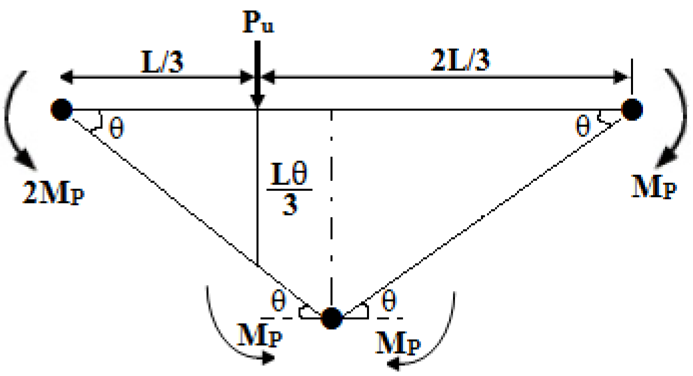
Work done by the load = $P_u\ (2L\theta/3)$

Work absorbed by the hinges = $2M_P\ \varphi + 2M_P\ (\theta + \varphi) + M_P\theta = 11M_P\theta$

For equilibrium, $P_u\ (2L\theta/3) = 11M_P\theta$

» $P_u = 16.5M_P/L$ ---(A)

In the second mechanism, hinges may form at X, Y and Z. The hinge at Y will work corresponding to the least moment of resistance at Y, i.e. M_P.



For equilibrium, $P_u\,(L\theta/3) = 2M_P\theta + M_P\theta + M_P\theta + M_P\theta$

» $P_u = 15M_P/L$ ---(B)

The mechanism is one which gives minimum of the collapse load (From (A) & (B)).

Hence $P_u = 15M_P/L$

37. A 588 cm^3 volume of moist sand weighs 1010 gm. Its dry weight is 918 gm and specific gravity of solids, G is 2.67. Assuming density of water as 1 gm/cm^3, the void ratio is _________.

Ans 0.71

- Dry density, γ_d = 918/588 = 1.56 gm/cm^3

Now, $\gamma_d = \dfrac{G\,\gamma_w}{1+e}$

» $1.56 = \dfrac{2.67 \times 1}{1+e}$

» e = 0.71

38. A 4 m thick layer of normally consolidated clay has an average void ratio of 1.30. Its compression index is 0.6 and coefficient of consolidation is 1 m^2/yr. If the increase in vertical pressure due to foundation load on the clay layer is equal to the existing effective overburden pressure, the change in the thickness of the clay layer is__________mm.

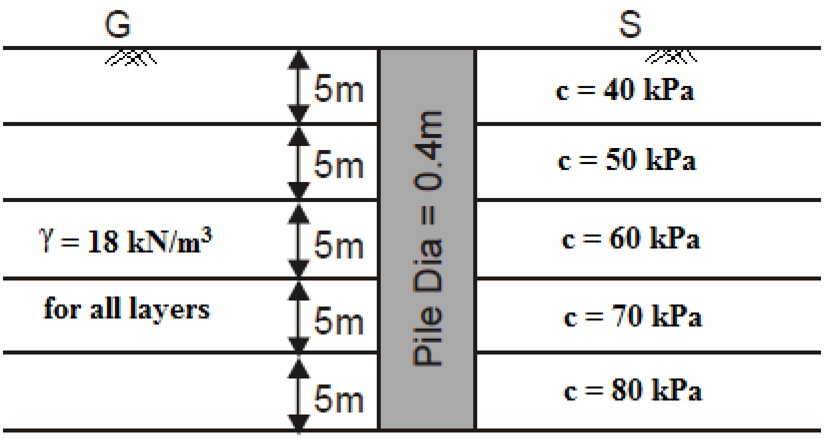
Ans 314.12

- Change in thickness = Settlement (S) = $\frac{C_c H_0}{1+e_0}\log\left(\frac{\bar{\sigma}_0+\Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$

  Here, $C_c = 0.6$, $e_0 = 1.30$, $H_0 = 4$ m, $\Delta\bar{\sigma} = \bar{\sigma}_0$

  » $S = \frac{0.6 \times 4}{1+1.3}\log(2) = 0.31412$ m= **314.12 mm**

39. A pile of diameter 0.4 m is fully embedded in a clay stratum having 5 layers, each 5 m thick as shown in the figure below. Assume a constant unit weight of soil as 18 kN/m^3 for all the layers. Using λ-method (λ = 0.15 for 25 m embedment length) and neglecting the end bearing component, the ultimate pile capacity (in kN) is __________.



Ans 1625.77 kN

- The ultimate pile capacity is given by,

  $Q_u = f_s A_s$ (where f_s is average unit skin friction and A_s is friction area)

  $f_s = \lambda\,(\bar{\sigma}_v + 2c)$ (where λ – friction capacity factor; $\bar{\sigma}_v$ = mean effective vertical stress for the embedment length; c – undrained cohesion)

  Here, $\bar{\sigma}_v = (1/2)\,(18 \times 5 + 18 \times 5 + 18 \times 5 + 18 \times 5 + 18 \times 5) = 225$ kN/m^2

  $c = (40 \times 5 + 50 \times 5 + 60 \times 5 + 70 \times 5 + 80 \times 5) / 25 = 60$ kPa = 60 kN/m^2

  Hence, $Q_u = 0.15\,(225 + 2 \times 60) \times \pi \times 0.4 \times 25 =$ **1625.77 kN**

40. Stress path equation for tri-axial test upon application of deviatoric stress is,

$q = 10\sqrt{3} + 0.5\,p$. The respective values of cohesion, c (in kPa) and angle of internal friction, ϕ are :

(A) 20 and 20^0

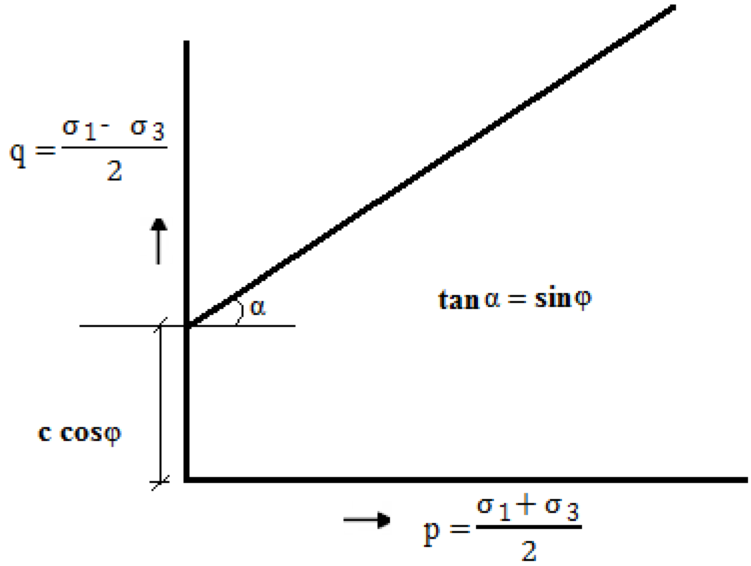
(B) 20 and 30^0

(C) 30 and 30^0

(D) 30 and 20^0

Ans (B) 20 and 30^0

- Let σ_1 and σ_3 are principal stresses. The plot for failure envelope between $p = (\sigma_1 + \sigma_3) / 2$ and $q = (\sigma_1 - \sigma_3) / 2$ is shown below:



The equation of the failure line is: $q = c\cos\phi + p\tan\alpha = c\cos\phi + p\sin\phi$

Given equation is: $q = 10\sqrt{3} + 0.5\,p$

On comparision, $\sin\phi = 0.5$ and $c\cos\phi = 10\sqrt{3}$

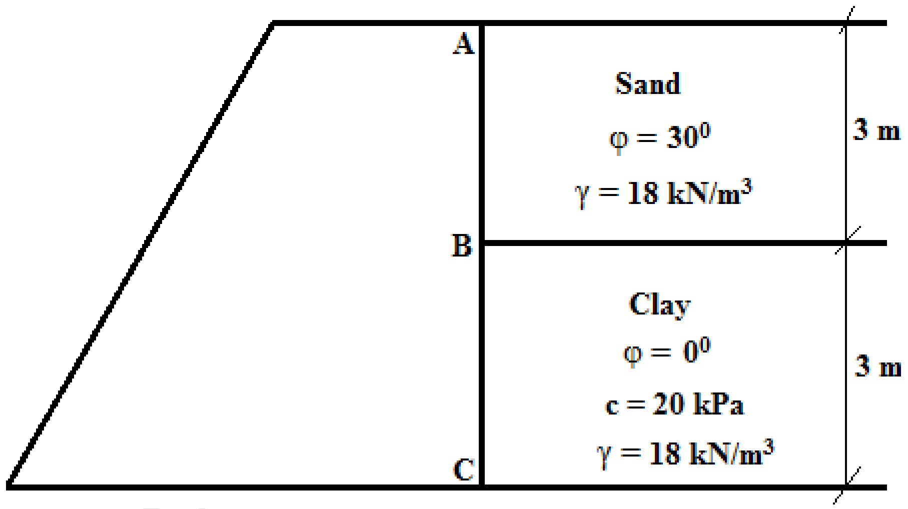
» $\phi = \sin^{-1}(0.5) = 30^0$ and

» $c = 10\sqrt{3} / \cos 30^0 = 20$ kPa

41. A 6 m high retaining wall having a smooth vertical back face retains a layered horizontal backfill. Top 3 m thick layer of the backfill is sand having an angle of internal friction, $\phi = 30^0$ while the bottom layer is 3 m thick clay with cohesion, c = 20 kPa. Assume unit weight for both sand and clay as 18 kN/m^3. The total active earth pressure per unit length of the wall (in kN/m) is:
(A) 150
(B) 216
(C) 156
(D) 196

Ans (A) 150

- The given data can be represented in the diagram below:



For sand layer:

Coefficient of active Earth Pressure, $K_{as} = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\sin 30^0}{1+\sin 30^0} = 1/3$

For clay layer:

Coefficient of active Earth Pressure, $K_{ac} = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\sin 0^0}{1+\sin 0^0} = 1$

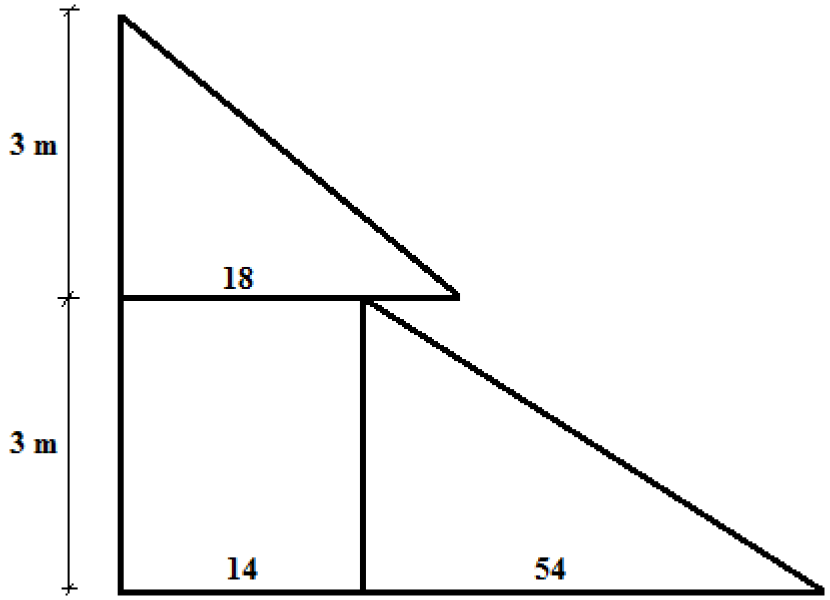
At A, Pressure (P_a) = 0

At B, $P_a = K_{as}\gamma(3) = (1/3) \times 18 \times 3 = 18$ kN/m^2

Below the interface, $P_a = K_{ac}\gamma(3) - 2c\sqrt{K_{ac}} = 1 \times 18 \times 3 - 2 \times 20 \times \sqrt{1} = 14$ kN/m^2

At C, $P_a = K_{ac}\gamma(6) - 2c\sqrt{K_{ac}} = 1 \times 18 \times 6 - 2 \times 20 \times \sqrt{1} = 68$ kN/m^2

The pressure diagram is shown below:



Hence, active earth pressure/wall, P = Area of pressure diagram

» P = (1/2) x 3 x 18 + 14 x 3 + (1/2) x 3 x 54 = **150 kN/m**

42. A field channel has cultivable commanded area of 2000 hectares. The intensities of irrigation for gram and wheat are 30% and 50% respectively. Gram has a kor period of 18 days, kor depth of 12 cm, while wheat has a kor period of 18 days and a kor depth of 15 cm. The discharge (in m^3/s) required in the field channel to supply water to the commanded area during the kor period is ________.

Ans 1.427 m^3/s

- Gram and wheat both are Rabi crops. The total discharge will be the sum of the discharges of these two crops.

  Area to be irrigated under gram = (30/100) x 2000 = 600 hectares

  Area to be irrigated under wheat = (50/100) x 2000 = 1000 hectares

  Duty for gram = 864 B / Δ = 864 x 18 / 12 = 1296 hec/cumec

  » Discharge required for gram = 600 / 1296 = 0.463 cumec (m^3/s)

  Duty for wheat = 864 B / Δ = 864 x 18 / 15 = 1036.8 hec/cumec

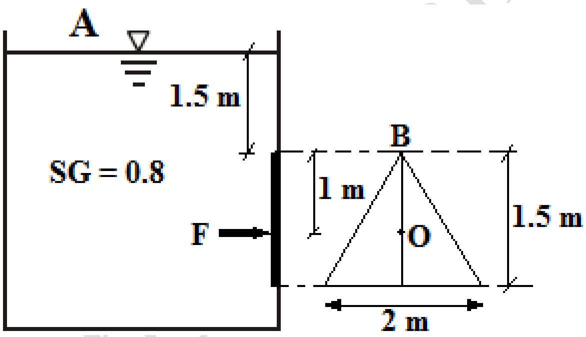
» Discharge required for wheat = 1000 / 1036.8 = 0.964 cumec (m^3/s)

Hence discharge required during kor period = 0.463 + 0.964 = **1.427 m^3/s**

43. A triangular gate with a base width of 2 m and a height of 1.5 m lies in a vertical plane. The top vertex of the gate is 1.5 m below the surface of a tank which contains oil of specific gravity 0.8. Considering the density of water and acceleration due to gravity to be 1000 kg/m^3 and 9.81 m/s^2 respectively, the hydrostatic force (in kN) exerted by the oil on the gate is ___________.

Ans 29.43 kN

- The given conditions can be drawn as below:



Density of oil (ρ) = 0.8 x 1000 = 800 kg/m^3

C.G. of triangular gate from upper vertex B = 2 x 1.5 / 3 = 1 m

Pressure at C.G. = ρgh = 800 x 9.81 x (1.5 + 1) = 19620 N/m^2 = 19.62 kN/m^2

Hence, hydrostatic force exerted by oil on gate,

F = Pressure at C.G. x Area of triangular gate = 19.62 x (1/2) x 2 x 1.5 = **29.43 kN**

44. The velocity components of a two dimensional plane motion of a fluid are :

$$u = \frac{y^3}{3} + 2x - x^2y \text{ and } v = xy^2 - 2y - \frac{x^3}{3}$$

The correct statement is:

(A) Fluid is incompressible and flow is irrotational

(B) Fluid is incompressible and flow is rotational

(C) Fluid is compressible and flow is irrotational

(D) Fluid is compressible and flow is rotational

Ans (A) Fluid is incompressible and flow is irrotational

- For any fluid V (u,v,w) to be incompressible, div V = 0 (where V = ui + vj + wk)

$$\text{div } V = \nabla . V = \left(\frac{i\,\partial}{\partial x} + \frac{j\,\partial}{\partial y} + \frac{k\,\partial}{\partial z}\right) . (ui + vj + wk) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Here, $u = \frac{y^3}{3} + 2x - x^2y$; $v = xy^2 - 2y - \frac{x^3}{3}$ and $w = 0$

$\frac{\partial u}{\partial x} = 2 - 2xy$; $\frac{\partial v}{\partial y} = 2xy - 2$;

» $\text{div } V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2 - 2xy + 2xy - 2 = 0$

Hence, the fluid is incompressible.

For flow to be irrotational, Curl V = 0

$$\text{Curl } V = \nabla \text{ X } V = \left(\frac{i\,\partial}{\partial x} + \frac{j\,\partial}{\partial y} + \frac{k\,\partial}{\partial z}\right) \text{ X } (ui + vj + wk)$$

$$= i\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + j\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + k\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

As this a 2-dimensional flow with w = 0, the condition for irrotational flow will be

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

Here, $\frac{\partial u}{\partial y} = y^2 - x^2$; $\frac{\partial v}{\partial x} = y^2 - x^2$;

» $\text{Curl } V = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = y^2 - x^2 - (y^2 - x^2) = 0$

Hence, the flow is irrotational.

45. The average surface area of a reservoir in the month of June is 20 km^2. In the same month, the average rate of inflow is 10 m^3/s, outflow rate is 15 m^3/s, monthly rainfall is 10 cm, monthly seepage loss is 1.8 cm and the storage change is 16 million m^3. The evaporation (in cm) in that month is :

(A) 46.8

(B) 136.0

(C) 13.6

(D) 23.4

Ans (D) 23.4

- Using Water-Budget method, the storage equation for Evaporation is

$P + V_{is} + V_{ig} = E + V_{os} + V_{og} + \Delta S + T_L$ ---------------------------(A)

where P = Precipitation; V_{is} = surface inflow; V_{ig} = groundwater inflow; E = evaporation; V_{os} = surface outflow; V_{og} = Seepage outflow; ΔS = increase in lake water storage; T_L = Transpiration Loss

Inflow:

P = 10 cm

$V_{is} = (10 \times 30 \times 24 \times 60 \times 60) / (20 \times 10^6) = 1.296$ m = 129.6 cm

Outflow:

$V_{os} = (15 \times 30 \times 24 \times 60 \times 60) / (20 \times 10^6) = 1.944$ m = 194.4 cm

V_{og} = 1.8 cm

As the outflow is more than inflow, ΔS will be negative i.e. decrease in level.

$\Delta S = (-16 \times 10^6) / (20 \times 10^6) = -0.08$ m = - 80 cm

Putting all values in equation (A), we get

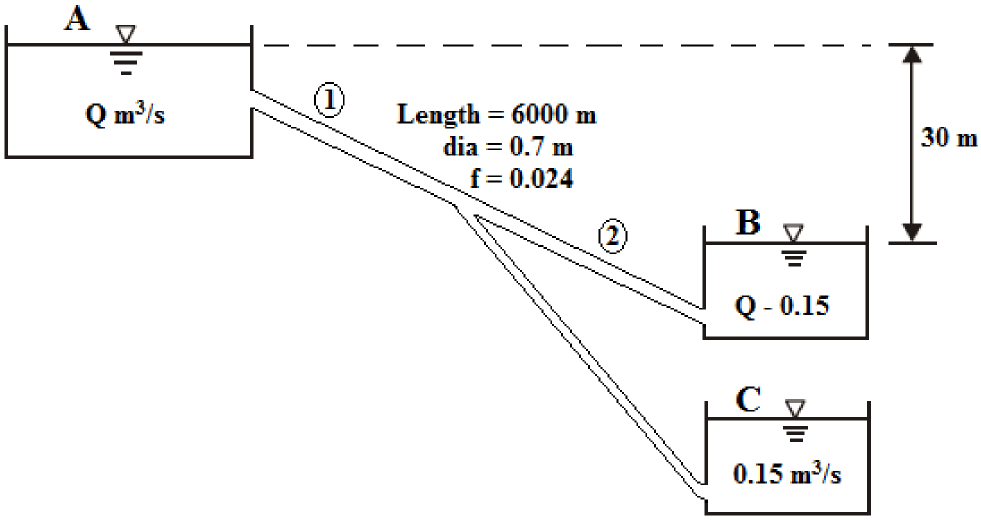
10 + 129.6 + 0 = E + 194.4 + 1.8 – 80 + 0

» E = 23.4 cm

46. A pipe of 0.7 m diameter has a length of 6 km and connects two reservoirs A and B. The water level in reservoir A is at an elevation 30 m above the water level in reservoir B. Halfway along the pipe line, there is a branch through which water can be supplied to a third reservoir C. The friction factor of the pipe is 0.024. The quantity of water discharged into reservoir C is 0.15 m^3/s. Considering the acceleration due to gravity as 9.81 m/s^2 and neglecting minor losses, the discharge (in m^3/s) into the reservoir B is _______________.

Ans 0.572

- The given conditions can be represented as below:



Applying Bernoulli's Equation at A and B,

Energy at A – Energy Lost = Energy at B

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A - \text{Energy Lost} = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B \text{ ------------------------------(A)}$$

Here, $P_A = P_B$; $V_A = V_B = 0$ (negligible); $Z_A - Z_B = 30$;

Putting all values in equation (A), we get

Energy Lost = head loss due to friction = $h_f = 30$

Further, $A = \pi\,(0.7)^2/4$; $V_1 = Q/A$; $V_2 = (Q - 0.15) / A$; $h_f = \frac{fLV^2}{2gd}$

» $V_1 = 2.598\,Q$ and $V_2 = 2.598\,(Q - 0.15)$

For part (1) of the pipe, $h_{f1} = \frac{fLV_1^2}{2gd} = \frac{0.024 \text{ x } 3000 \text{ x } (2.598\,Q)^2}{2 \text{ x } 9.81 \text{ x } 0.7}$

For part (2) of the pipe, $h_{f2} = \frac{fLV_2^2}{2gd} = \frac{0.024 \text{ x } 3000 \text{ x } [2.598\,(Q-0.15)]^2}{2 \text{ x } 9.81 \text{ x } 0.7}$

Now, $h_{f1} + h_{f2} = 30$

» $\frac{0.024 \text{ x } 3000 \text{ x } (2.598\,Q)^2}{2 \text{ x } 9.81 \text{ x } 0.7} + \frac{0.024 \text{ x } 3000 \text{ x } [2.598\,(Q-0.15)]^2}{2 \text{ x } 9.81 \text{ x } 0.7} = 30$

» $Q^2 + (Q - 0.15)^2 = 0.848$

» $Q = 0.722\ m^3/s$

Hence, discharge into reservoir B = Q – 0.15 = **$0.572\ m^3/s$**

47. A landfill is to be designed to serve a population of 200000 for a period of 25 years. The solid waste (SW) generation is 2 kg/person/day. The density of the un-compacted SW is 100 kg/m^3 and a compaction ratio of 4 is suggested. The ratio of compacted fill (i.e., SW+ cover) to compacted SW is 1.5. The landfill volume (in million m^3) required is __________.

Ans 13.69

- Population = 200000

  Rate of Solid waste (SW) generation = 2 kg/person/day

  » Total Solid waste (SW) generated/day = 2 x 200000 = 400000 kg/day

  Design life of landfill = 25 years

  » Total Solid waste (SW) to be dumped in landfill in 25 years = 400000 x 25 x 365 kg

  As the compaction ratio = 4

  The density of compacted SW = 100 x 4 = 400 kg/m^3

  Ratio of compacted fill (i.e., SW+ cover) to compacted SW = 1.5

  » Volume of compacted fill = 1.5 x volume of SW = 1.5 x (Mass/density)

  = 1.5 x 400000 x 25 x 365 / 400 = 13687500 m^3 = **13.69 million m^3**

48. A water treatment plant of capacity, 1 m^3/s has filter boxes of dimensions 6 m × 10 m. Loading rate to the filters is 120 m^3/day/m^2. When two of the filters are out of service for back washing, the loading rate (in m^3/day/m^2) is __________.

Ans 144

- Capacity of water treatment plant = 1 m^3/s

  Loading rate = 120 m^3/day/m^2 = 120/(24 x 60 x 60) m^3/day/m^2 = 1.389 x 10^{-3} m^3/s/m^2

  Area of each filter = 6 x 10 = 60 m^2

  Let total no. of filters = n

  Loading rate x Area x n = Capacity

  » n = 1 / (60 x 1.389 x 10^{-3}) = 12

  When two of the filters are out of service for back washing,

  The no. of filters in working condition = 12 – 2 = 10

Hence loading rate = Capacity / (Area x 10) = 1 / (10 x 60) $m^3/s/m^2$

= 24 x 60 x 60 / (10 x 60) $m^3/day/m^2$ = 144 $m^3/day/m^2$

49. Ultimate BOD of a river water sample is 20 mg/L. BOD rate constant (natural log) is 0.15 day^{-1}. The respective values of BOD (in %) exerted and remaining after 7 days are :

(A) 45 and 55

(B) 55 and 45

(C) 65 and 35

(D) 75 and 25

Ans (C) 65 and 35

- The formula for BOD is, $L_t = L_0\ (1 - e^{-kt})$

Here, L_0 = 20 mg/L; k = 0.15 day^{-1}

» $L_7 = 20\ (1 - e^{-0.15 \times 7}) = 13$

Hence, BOD (in %) exerted after 7 days = $\frac{13}{20}$ x 100 = **65%**

BOD (in %) remaining after 7 days = 100 – 65 = **35%**

50. In a wastewater treatment plant, primary sedimentation tank (PST) designed at an overflow rate of 32.5 $m^3/day/m^2$ is 32.5 m long, 8.0 m wide and liquid depth of 2.25 m. If the length of the weir is 75 m, the weir loading rate (in m^3/day/m) is ______.

Ans 112.67

- Overflow rate = Flow / Area

Here, Overflow rate = 32.5 $m^3/day/m^2$; Area = 32.5 X 8 = 260 m^2

» Flow = 32.5 X 260 = 8450 m^3/day

Hence weir loading = Flow / length of weir = 8450 / 75 = **112.67 m^3/day/m**

51. The relation between speed u (in km/h) and density k (number of vehicles/km) for a traffic stream on a road is u = 70 – 0.7k. The capacity on this road is __________ vph (vehicles/hour).

Ans 1750

- Given, u = 70 – 0.7k

  The maximum flow speed, u_f will be obtained when density (k) = 0.

  » u_f = 70 km/h

  Jam density will be obtained, when the flow speed (u) = 0

  » 0 = 70 – 0.7k_j

  k_j = 100 veh/km

  Hence, Capacity (q) = u_f k_j / 4 = 1750 vph (veh/hour)

52. Match the information related to tests on aggregates given in Group-I with that in Group-II

Group-I	Group-II
P. Resistance to impact	1. Hardness
Q. Resistance to wear	2. Strength
R. Resistance to weathering action	3. Toughness
S. Resistance to crushing	4. Soundness

(A) P-1, Q-3, R-4, S-2

(B) P-3, Q-1, R-4, S-2

(C) P-4, Q-1, R-3, S-2

(D) P-3, Q-4, R-2, S-1

Ans (B) P-3, Q-1, R-4, S-2

- **1) Strength – Resistance to crushing – Aggregate Crushing Value (ACV).**

  Generally, this test is made on single sized aggregate passing **12.5 mm** and retained on **10 mm** sieve. The aggregate is placed in a cylindrical mould and a load of **40 ton** is applied through a plunger. The material crushed to finer than **2.36 mm** is separated and expressed as a percentage of the original weight taken in the mould. This percentage is referred as **ACV**. The crushing value of aggregate is restricted to **30%** for concrete used for roads and pavements and **45%** may be permitted for other structures.

When ACV become 30 or higher, the result is likely to be inaccurate, in which case the aggregate should be subjected to **"10% fines value"** test which gives a better picture about the strength of such aggregates.

2) Toughness – Resistance to Impact – Aggregate Impact Value (AIV).

Aggregate sample kept in a mould is subjected to **15 blows** of a metal hammer of weight **14 kg** falling from a height of **38 cm**. The ratio of the weight of fines (finer than **2.36 mm** size) formed, to the weight of the total sample taken is expressed as a percentage. This is known as **AIV**. **IS 283-1970** specifies that AIV shall not exceed **45%** by weight for aggregate used for concrete other than wearing surface and **30%** by weight, for concrete for wearing surfaces, such as runways, roads and pavements.

3) Hardness – Resistance to wear – Aggregate Abrasion Value (AAV).

The abrasion value should not be more than **30%** for wearing surfaces and not more than **50%** for concrete other than wearing surface.

4) Soundness – Resistance to weathering action

Soundness refers to the ability of aggregate to resist excessive changes in volume as a result of changes in physical conditions. Soundness test is specified in **IS 2386 (Part V)**. As a general guide, it can be taken that the average loss of weight after **10 cycles** should not exceed **12%** and **18%** when tested with sodium sulphate and magnesium sulphate respectively.

53. In Marshall method of mix design, the coarse aggregate, fine aggregate, fines and bitumen having respective values of specific gravity 2.60, 2.70, 2.65 and 1.01, are mixed in the relative proportions (% by weight) of 55.0, 35.8, 3.7 and 5.5 respectively. The theoretical specific gravity of the mix and the effective specific gravity of the aggregates in the mix respectively are :

(A) 2.42 and 2.63

(B) 2.42 and 2.78

(C) 2.42 and 2.93

(D) 2.64 and 2.78

Ans (A) 2.42 and 2.63

- Theoretical specific gravity of the mix (G_m) = $\dfrac{P_{ca}+P_{fa}+P_f+P_b}{\frac{P_{ca}}{G_{ca}}+\frac{P_{fa}}{G_{fa}}+\frac{P_f}{G_f}+\frac{P_b}{G_b}}$

» $G_m = \dfrac{55+35.8+3.7+5.5}{\frac{55}{2.60}+\frac{35.8}{2.70}+\frac{3.7}{2.65}+\frac{5.5}{1.01}} =$ **2.42**

Effective specific gravity of the aggregates (G_{se}) = $\dfrac{100-P_b}{\frac{100}{G_m}-\frac{P_b}{G_b}}$

» $G_{se} = \dfrac{100-5.5}{\frac{100}{2.42}-\frac{5.5}{1.01}} =$ **2.63**

54. The bearings of two inaccessible stations, S_1 (Easting 500 m, Northing 500 m) and S_2 (Easting 600 m, Northing 450 m) from a station S_3 were observed as 225^0 and $153^026'$ respectively. The independent Easting (in m) of station S_3 is:

(A) 450.000

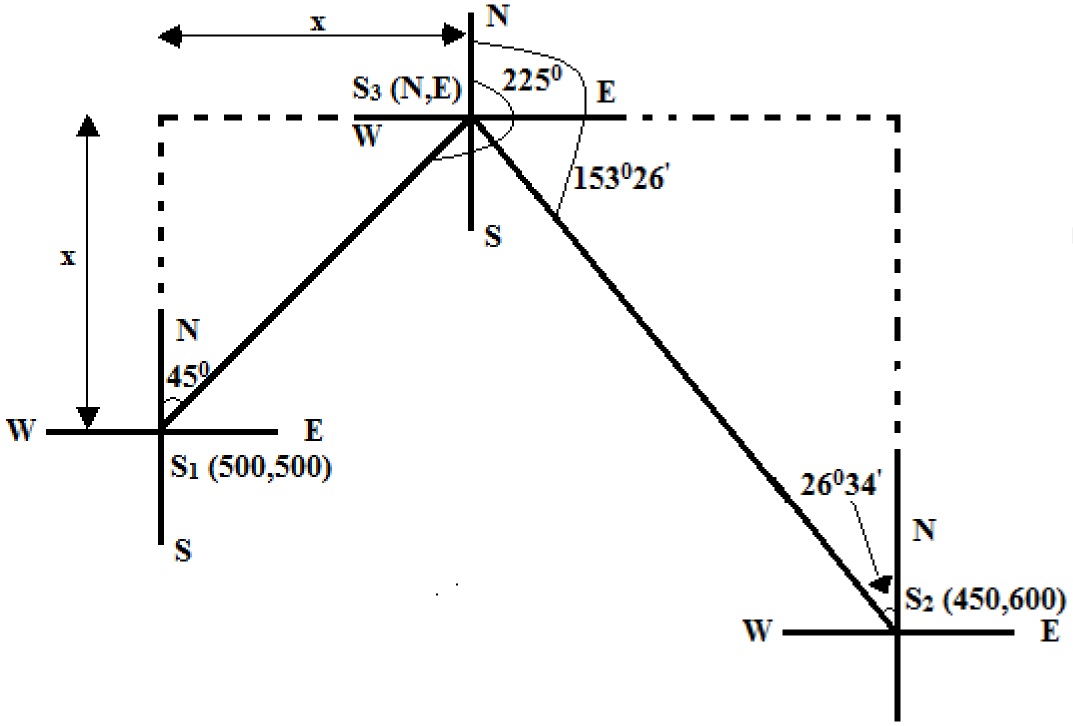
(B) 570.710

(C) 550.000

(D) 650.000

Ans (C) 550.000

- The given conditions can be shown as below:



From the figure, as the angle is 45^0

N = 500 + x; E = 500 + x

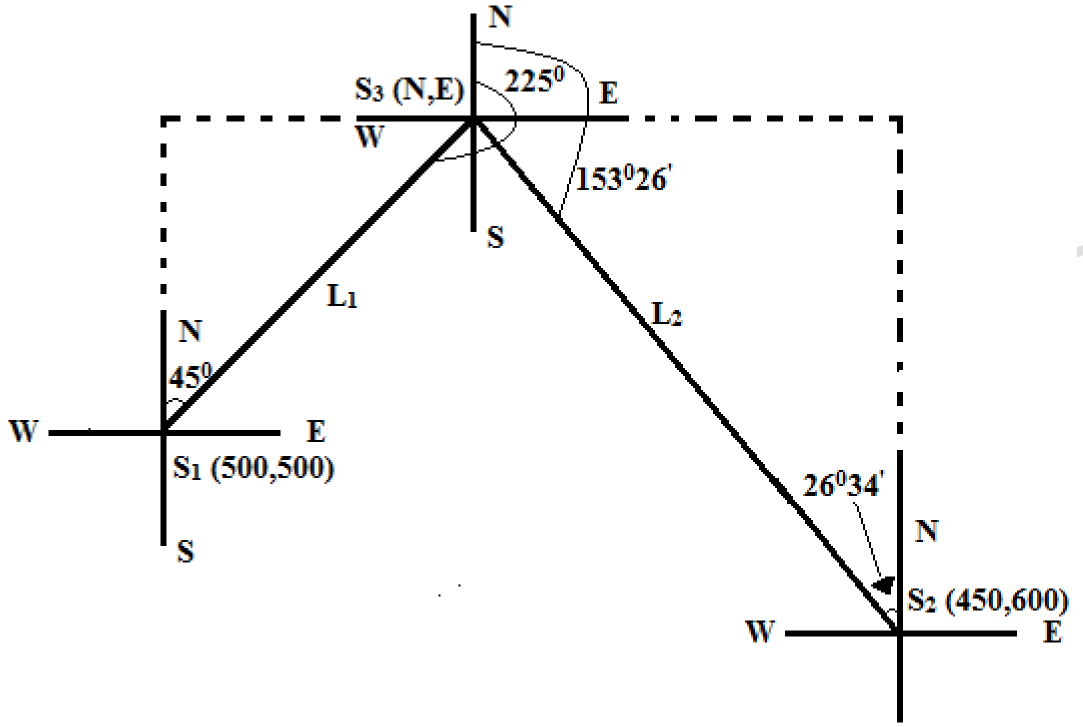
tan $26^0 34'$ = (600 – E) / (N – 450) = (100 – x) / (50 + x)

0.5 = (100 – x) / (50 + x)

» x = 50

Hence, Easting of S_3 = 500 + 50 = 550

- The question can be solved as below also



Northing of S_3 from both the triangles,

$500 + L_1\cos45^0 = 450 + L_2\cos26^034'$

$0.894\ L_2 - 0.707\ L_1 = 50$ --------------------------------------(A)

Easting of S_3 from both the triangles,

$500 + L_1\sin45^0 = 600 - L_2\sin26^034'$

$0.707\ L_1 + 0.447\ L_2 = 100$ -----------------------------------(B)

From equations (A) and (B), we get

$L_1 = 70.7$; $L_2 = 111.86$

Hence, Easting of $S_3 = 500 + L_1\cos45^0 = 550$

55. Two Pegs A and B were fixed on opposite banks of a 50 m wide river. The level was set up at A and the staff readings on Pegs A and B were observed as 1.350 m and 1.550 m, respectively. Thereafter the instrument was shifted and set up at B. The staff readings on Pegs B and A were observed as 0.750 m and 0.550 m, respectively. If the R.L. of Peg A is 100.200 m, the R.L. (in m) of Peg B is _________.

Ans 100.00 m

- The given data can be arranged as below:

Level at	Reading on Peg A (m)	Reading on Peg B (m)
A	1.350	1.550
B	0.550	0.750

Clearly, from the table it can be deciphered that the R.L. of A is more than R.L of B.

Change in R.L. = $h_{AB} = \frac{(B_1 - A_1) + (B_2 - A_2)}{2} = \frac{(1.55 - 1.35) + (0.75 - 0.55)}{2} = 0.2$ m

» R.L. of B = R.L. of A – h_{AB} = 100.200 – 0.200 = **100.00 m**