

**NCERT Solutions for Class 9<sup>th</sup> Mathematics**

**Chapter 2 – POLYNOMIALS**

**EXERCISE 2.1**

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i)  $4x^2 - 3x + 7$       (ii)  $y^2 + \sqrt{2}$       (iii)  $3\sqrt{t} + t\sqrt{2}$       (iv)  $y + \frac{2}{y}$       (v)  $x^{10} + y^3 + t^{50}$

**Ans** (i)  $4x^2 - 3x + 7$  is a polynomial in one variable as there is only one variable i.e. 'x' and its a polynomial because power or exponent of x (2,1,0) in each term is a whole number.

(ii)  $y^2 + \sqrt{2}$  is a polynomial in one variable as there is only one variable i.e. 'y' and its a polynomial because power or exponent of y (2,0) in each term is a whole number.

(iii)  $3\sqrt{t} + t\sqrt{2}$  is not a polynomial in one variable as power or exponent of variable 't' in first term is 1/2, which is not a whole number.

(iv)  $y + \frac{2}{y}$  is not a polynomial in one variable as power or exponent of variable 'y' in second term is -1, which is not a whole number.

(v)  $x^{10} + y^3 + t^{50}$  is not a polynomial in one variable but it is a polynomial in 3 variables (x, y, t).

2. Write the coefficients of  $x^2$  in each of the following:

(i)  $2 + x^2 + x$       (ii)  $2 - x^2 + x^3$       (iii)  $\frac{\pi}{2}x^2 + x$       (iv)  $\sqrt{2}x - 1$

**Ans** **Coefficient** of any term is the constant multiplying with the variable.

(i)  $2 + x^2 + x$

Coefficient of  $x^2$  is 1.

(ii)  $2 - x^2 + x^3$

Coefficient of  $x^2$  is -1.

(iii)  $\frac{\pi}{2}x^2 + x$

Coefficient of  $x^2$  is  $\pi/2$ .

(iv)  $\sqrt{2}x - 1$

Coefficient of  $x^2$  is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

**Ans** Binomial (2 terms only) of degree 35 is  $x^{35} - 7$

Monomial (1 term only) of degree 100 is  $3x^{100}$ .

4. Write the degree of each of the following polynomials:

(i)  $5x^3 + 4x^2 + 7x$       (ii)  $4 - y^2$       (iii)  $5t - \sqrt{7}$       (iv) 3

**Ans Degree of polynomial** = Highest power of variable

(i)  $5x^3 + 4x^2 + 7x$

**Degree = 3** (because highest power of  $x$  is 3)

(ii)  $4 - y^2$

**Degree = 2** (because highest power of  $y$  is 2)

(iii)  $5t - \sqrt{7}$

**Degree = 1** (because highest power of  $t$  is 1)

(iv) 3

**Degree = 0** (because there is no variable i.e. it is a constant polynomial)

5. Classify the following as linear, quadratic and cubic polynomials:

(i)  $x^2 + x$       (ii)  $x - x^3$       (iii)  $y + y^2 + 4$       (iv)  $1 + x$

(v)  $3t$       (vi)  $r^2$       (vii)  $7x^3$

**Ans Linear polynomial:** It has degree of 1.

**Quadratic polynomial:** It has degree of 2.

**Cubic polynomial:** It has degree of 3.

(i)  $x^2 + x$

**Quadratic polynomial** (because degree of polynomial is 2)

(ii)  $x - x^3$

**Cubic polynomial** (because degree of polynomial is 3)

(iii)  $y + y^2 + 4$

**Quadratic polynomial** (because degree of polynomial is 2)

(iv)  $1 + x$

**Linear polynomial** (because degree of polynomial is 1)

(v)  $3t$

**Linear polynomial** (because degree of polynomial is 1)

(vi)  $r^2$

**Quadratic polynomial** (because degree of polynomial is 2)

(vii)  $7x^3$

**Cubic polynomial** (because degree of polynomial is 3)

### EXERCISE 2.2

1. Find the value of the polynomial  $5x - 4x^2 + 3$  at

(i)  $x = 0$       (ii)  $x = -1$       (iii)  $x = 2$

**Ans** (i)  $p(x) = 5x - 4x^2 + 3$

The value of the polynomial at  $x = 0$  is

$$p(0) = 5(0) - 4(0)^2 + 3 = \mathbf{3}$$

(ii)  $p(x) = 5x - 4x^2 + 3$

The value of the polynomial at  $x = -1$  is

$$p(-1) = 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = \mathbf{-6}$$

(iii)  $p(x) = 5x - 4x^2 + 3$

The value of the polynomial at  $x = 2$  is

$$p(2) = 5(2) - 4(2)^2 + 3 = 10 - 16 + 3 = \mathbf{-3}$$

2. Find  $p(0)$ ,  $p(1)$  and  $p(2)$  for each of the following polynomials:

(i)  $p(y) = y^2 - y + 1$

(ii)  $p(t) = 2 + t + 2t^2 - t^3$

(iii)  $p(x) = x^3$

$$(iv) p(x) = (x - 1)(x + 1)$$

**Ans** (i)  $p(y) = y^2 - y + 1$

$$p(0) = (0)^2 - (0) + 1 = 1$$

$$p(1) = (1)^2 - (1) + 1 = 1$$

$$p(2) = (2)^2 - (2) + 1 = 3$$

$$(ii) p(t) = 2 + t + 2t^2 - t^3$$

$$p(0) = 2 + (0) + 2(0)^2 - (0)^3 = 2$$

$$p(1) = 2 + (1) + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$$

$$p(2) = 2 + (2) + 2(2)^2 - (2)^3 = 2 + 2 + 8 - 8 = 4$$

$$(iii) p(x) = x^3$$

$$p(0) = (0)^3 = 0$$

$$p(1) = (1)^3 = 1$$

$$p(2) = (2)^3 = 8$$

$$(iv) p(x) = (x - 1)(x + 1)$$

$$p(0) = (0 - 1)(0 + 1) = -1 \times 1 = -1$$

$$p(1) = (1 - 1)(1 + 1) = 0 \times 2 = 0$$

$$p(2) = (2 - 1)(2 + 1) = 1 \times 3 = 3$$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

$$(i) p(x) = 3x + 1, x = -1/3$$

$$(ii) p(x) = 5x - \pi, x = 4/5$$

$$(iii) p(x) = x^2 - 1, x = 1, -1$$

$$(iv) p(x) = (x + 1)(x - 2), x = -1, 2$$

$$(v) p(x) = x^2, x = 0$$

$$(vi) p(x) = lx + m, x = -m/l$$

$$(vii) p(x) = 3x^2 - 1, x = -1/\sqrt{3}, 2/\sqrt{3}$$

$$(viii) p(x) = 2x + 1, x = 1/2$$

**Ans** If  $x = a$  is a zero of the polynomial  $p(x)$ , then at  $x = a$  the value of polynomial will be zero i.e.  $p(a) = 0$ .

$$(i) p(x) = 3x + 1, x = -1/3$$

$$\gg p(-1/3) = 3(-1/3) + 1 = -1 + 1 = 0$$

Hence  $x = -1/3$  is a zero of the given polynomial.

(ii)  $p(x) = 5x - \pi$ ,  $x = 4/5$

»  $p(4/5) = 5(4/5) - \pi = 4 - \pi \neq 0$

Hence  $x = 4/5$  is not a zero of the given polynomial.

(iii)  $p(x) = x^2 - 1$ ,  $x = 1, -1$

»  $p(1) = (1)^2 - 1 = 1 - 1 = 0$

$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$

Hence  $x = 1, -1$  are zeroes of the given polynomial.

(iv)  $p(x) = (x + 1)(x - 2)$ ,  $x = -1, 2$

»  $p(-1) = (-1 + 1)(-1 - 2) = 0 \times -3 = 0$

$p(2) = (2 + 1)(2 - 2) = 3 \times 0 = 0$

Hence  $x = -1, 2$  are zeroes of the given polynomial.

(v)  $p(x) = x^2$ ,  $x = 0$

»  $p(0) = (0)^2 = 0$

Hence  $x = 0$  is a zero of the given polynomial.

(vi)  $p(x) = lx + m$ ,  $x = -m/l$

»  $p(-m/l) = l(-m/l) + m = -m + m = 0$

Hence  $x = -m/l$  is a zero of the given polynomial.

(vii)  $p(x) = 3x^2 - 1$ ,  $x = -1/\sqrt{3}, 2/\sqrt{3}$

»  $p(-1/\sqrt{3}) = 3(-1/\sqrt{3})^2 - 1 = 1 - 1 = 0$

$p(2/\sqrt{3}) = 3(2/\sqrt{3})^2 - 1 = 4 - 1 = 3$

Hence  $x = -1/\sqrt{3}$  is a zero, but  $x = 2/\sqrt{3}$  is not a zero of the given polynomial.

(viii)  $p(x) = 2x + 1$ ,  $x = 1/2$

»  $p(1/2) = 2(1/2) + 1 = 1 + 1 = 2$

Hence  $x = 1/2$  is not a zero of the given polynomial.

4. Find the zero of the polynomial in each of the following cases:

(i)  $p(x) = x + 5$

(ii)  $p(x) = x - 5$

(iii)  $p(x) = 2x + 5$

(iv)  $p(x) = 3x - 2$

(v)  $p(x) = 3x$

(vi)  $p(x) = ax, a \neq 0$

(vii)  $p(x) = cx + d, c \neq 0, c, d$  are real numbers.

**Ans** The zero of any polynomial  $p(x)$  can be found by using equation  $p(x) = 0$ .

(i)  $p(x) = x + 5$

For finding zero of the polynomial, put  $p(x) = 0$

»  $x + 5 = 0$

»  **$x = -5$**

Thus,  $x = -5$  is the zero of the given polynomial.

(ii)  $p(x) = x - 5$

For finding zero of the polynomial, put  $p(x) = 0$

»  $x - 5 = 0$

»  **$x = 5$**

Thus,  $x = 5$  is the zero of the given polynomial.

(iii)  $p(x) = 2x + 5$

For finding zero of the polynomial, put  $p(x) = 0$

»  $2x + 5 = 0$

»  **$x = -5/2$**

Thus,  $x = -5/2$  is the zero of the given polynomial.

(iv)  $p(x) = 3x - 2$

For finding zero of the polynomial, put  $p(x) = 0$

»  $3x - 2 = 0$

»  **$x = 2/3$**

Thus,  $x = 2/3$  is the zero of the given polynomial.

(v)  $p(x) = 3x$

For finding zero of the polynomial, put  $p(x) = 0$

»  $3x = 0$

»  **$x = 0$**

Thus,  $x = 0$  is the zero of the given polynomial.

(vi)  $p(x) = ax, a \neq 0$

For finding zero of the polynomial, put  $p(x) = 0$

»  $ax = 0$

»  **$x = 0$  (because  $a \neq 0$ )**

Thus,  $x = 0$  is the zero of the given polynomial.

(vii)  $p(x) = cx + d, c \neq 0, c, d$  are real numbers

For finding zero of the polynomial, put  $p(x) = 0$

»  $cx + d = 0$

»  **$x = -d/c$**

Thus,  $x = -d/c$  is the zero of the given polynomial.

**EXERCISE 2.3**

1. Find the remainder when  $x^3 + 3x^2 + 3x + 1$  is divided by  
 (i)  $x + 1$       (ii)  $x - 1/2$       (iii)  $x$       (iv)  $x + \pi$       (v)  $5 + 2x$

**Ans** (i)  $x + 1$

By long division,

$$\begin{array}{r}
 \phantom{x^3 + 3x^2 + 3x + 1} \cdot \phantom{+} x^2 + 2x + 1 \\
 \hline
 x + 1 \overline{) x^3 + 3x^2 + 3x + 1} \\
 \underline{x^3 + x^2} \phantom{+ 3x + 1} \\
 2x^2 + 3x + 1 \\
 \underline{2x^2 + 2x} \phantom{+ 1} \\
 x + 1 \\
 \underline{x + 1} \\
 0
 \end{array}$$

Hence, the remainder = 0.

By remainder theorem,

$p(x) = x^3 + 3x^2 + 3x + 1$  and zero of  $x + 1$  is -1 (obtained by putting  $x + 1 = 0$ ).

So,  $p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = 0$

Hence, the remainder = 0.



(ii)  $x - 1/2$

By long division,

$$\begin{array}{r}
 x^2 + \frac{7}{2}x + \frac{19}{4} \\
 \hline
 x - \frac{1}{2} \overline{) \begin{array}{l} x^3 + 3x^2 + 3x + 1 \\ x^3 - \frac{x^2}{2} \\ \hline \frac{7}{2}x^2 + 3x + 1 \\ \frac{7}{2}x^2 - \frac{7}{4}x \\ \hline \frac{19}{4}x + 1 \\ \frac{19}{4}x - \frac{19}{8} \\ \hline \frac{27}{8} \end{array} }
 \end{array}$$

Hence, the remainder =  $27/8$ .

By remainder theorem,

$p(x) = x^3 + 3x^2 + 3x + 1$  and zero of  $x - 1/2$  is  $1/2$  (obtained by putting  $x - 1/2 = 0$ ).

So,  $p(1/2) = (1/2)^3 + 3(1/2)^2 + 3(1/2) + 1 = 1/8 + 3/4 + 3/2 + 1 = 27/8$

Hence, the remainder =  $27/8$ .

(iii) x

By long division,

$$\begin{array}{r}
 \phantom{x} \cdot \phantom{+} x^2 + 3x + 3 \\
 x \overline{) \phantom{+} x^3 + 3x^2 + 3x + 1} \\
 \underline{x^3} \phantom{+ 3x^2 + 3x + 1} \\
 \phantom{x^3 + } 3x^2 + 3x + 1 \\
 \underline{3x^2} \phantom{+ 3x + 1} \\
 \phantom{x^3 + 3x^2 + } 3x + 1 \\
 \underline{3x} \phantom{+ 1} \\
 \phantom{x^3 + 3x^2 + 3x + } 1
 \end{array}$$

Hence, the remainder = 1.

By remainder theorem,

$p(x) = x^3 + 3x^2 + 3x + 1$  and zero of x is 0 (obtained by putting  $x = 0$ ).

So,  $p(0) = (0)^3 + 3(0)^2 + 3(0) + 1 = 1$

Hence, the remainder = 1.

(iv)  $x + \pi$

By long division,

$$\begin{array}{r}
 x^2 + (3 - \pi)x + (3 - 3\pi + \pi^2) \\
 x + \pi \overline{) \begin{array}{l} x^3 + 3x^2 + 3x + 1 \\ x^3 + \pi x^2 \\ \hline (3 - \pi)x^2 + 3x + 1 \\ (3 - \pi)x^2 + (3\pi - \pi^2)x \\ \hline (3 - 3\pi + \pi^2)x + 1 \\ (3 - 3\pi + \pi^2)x + (3 - 3\pi + \pi^2)\pi \\ \hline 1 - 3\pi + 3\pi^2 - \pi^3 \end{array} }
 \end{array}$$

Hence, the remainder =  $-\pi^3 + 3\pi^2 - 3\pi + 1$ .

By remainder theorem,

$p(x) = x^3 + 3x^2 + 3x + 1$  and zero of  $x + \pi$  is  $-\pi$  (obtained by putting  $x + \pi = 0$ ).

So,  $p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1 = -\pi^3 + 3\pi^2 - 3\pi + 1$

Hence, the remainder =  $-\pi^3 + 3\pi^2 - 3\pi + 1$ .

(v)  $5 + 2x$

By long division,

$$\begin{array}{r}
 \frac{x^2}{2} + \frac{x}{4} + \frac{7}{8} \\
 2x + 5 \overline{) \begin{array}{l} x^3 + 3x^2 + 3x + 1 \\ x^3 + \frac{5}{2}x^2 \\ \hline \frac{1}{2}x^2 + 3x + 1 \\ \frac{1}{2}x^2 + \frac{5}{4}x \\ \hline \frac{7}{4}x + 1 \\ \frac{7}{4}x + \frac{35}{8} \\ \hline -\frac{27}{8} \end{array}}
 \end{array}$$

Hence, the remainder =  $-27/8$

By remainder theorem,

$p(x) = x^3 + 3x^2 + 3x + 1$  and zero of  $2x + 5$  is  $-5/2$  (obtained by putting  $2x + 5 = 0$ ).

So,  $p(-5/2) = (-5/2)^3 + 3(-5/2)^2 + 3(-5/2) + 1 = -125/8 + 75/4 - 15/2 + 1 = -27/8$

Hence, the remainder =  $-27/8$ .

2. Find the remainder when  $x^3 - ax^2 + 6x - a$  is divided by  $x - a$ .

**Ans** By long division,

$$\begin{array}{r}
 x^2 + 6 \\
 x - a \overline{) x^3 - ax^2 + 6x - a} \\
 \underline{x^3 + ax^2} \phantom{+ 6x - a} \\
 6x - a \\
 \underline{6x - 6a} \\
 5a
 \end{array}$$

Hence, the remainder =  $5a$

**By remainder theorem,**

$p(x) = x^3 - ax^2 + 6x - a$  and zero of  $x - a$  is  $a$  (obtained by putting  $x - a = 0$ ).

So,  $p(a) = (a)^3 - a(a)^2 + 6(a) - a = 5a$

Hence, the remainder =  $5a$ .

3. Check whether  $7 + 3x$  is a factor of  $3x^3 + 7x$ .

**Ans**  $7 + 3x$  will be a factor of  $p(x) = 3x^3 + 7x$ , if it divides  $p(x)$  leaving remainder zero.

By applying remainder theorem to calculate the remainder, we have

$p(x) = 3x^3 + 7x$  and zero of  $7 + 3x$  is  $-7/3$  (obtained by putting  $7 + 3x = 0$ )

So,  $p(-7/3) = 3(-7/3)^3 + 7(-7/3) = -343/9 - 49/3 = -490/9$

Since the remainder is not zero,  $7 + 3x$  is not a factor of  $3x^3 + 7x$ .

### EXERCISE 2.4

1. Determine which of the following polynomials has  $(x + 1)$  a factor :

- (i)  $x^3 + x^2 + x + 1$                       (ii)  $x^4 + x^3 + x^2 + x + 1$   
 (iii)  $x^4 + 3x^3 + 3x^2 + x + 1$         (iv)  $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

**Ans** Zero of  $x + 1$  is  $-1$  (obtained by putting  $x + 1 = 0$ ).

$x + 1$  will be a factor of any polynomial  $p(x)$ , only if  $p(-1) = 0$ .

(i)  $p(x) = x^3 + x^2 + x + 1$

»  $p(-1) = (-1)^3 + (-1)^2 + (-1) + 1 = 0$

Hence,  $x + 1$  is a factor of the given polynomial.

(ii)  $p(x) = x^4 + x^3 + x^2 + x + 1$

»  $p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1 \neq 0$

Hence,  $x + 1$  is not a factor of the given polynomial.

(iii)  $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

»  $p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1 = 1 \neq 0$

Hence,  $x + 1$  is not a factor of the given polynomial.

(iv)  $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

»  $p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2} = -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = \sqrt{2} \neq 0$

Hence,  $x + 1$  is not a factor of the given polynomial.

2. Use the Factor Theorem to determine whether  $g(x)$  is a factor of  $p(x)$  in each of the following cases:

(i)  $p(x) = 2x^3 + x^2 - 2x - 1$ ,  $g(x) = x + 1$

(ii)  $p(x) = x^3 + 3x^2 + 3x + 1$ ,  $g(x) = x + 2$

(iii)  $p(x) = x^3 - 4x^2 + x + 6$ ,  $g(x) = x - 3$

**Ans** (i)  $p(x) = 2x^3 + x^2 - 2x - 1$ ,  $g(x) = x + 1$

Zero of  $g(x) = x + 1$  is  $-1$  (obtained by putting  $g(x) = 0$ )

According to Factor Theorem, if  $g(x) = x + 1$  is a factor of  $p(x)$ , then  $p(-1) = 0$ .

Now,  $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 = 0$

Thus, by the Factor Theorem,  $g(x) = x + 1$  is a factor of  $p(x) = 2x^3 + x^2 - 2x - 1$ .

(ii)  $p(x) = x^3 + 3x^2 + 3x + 1$ ,  $g(x) = x + 2$

Zero of  $g(x) = x + 2$  is -2 (obtained by putting  $g(x) = 0$ )

According to Factor Theorem, if  $g(x) = x + 2$  is a factor of  $p(x)$ , then  $p(-2) = 0$ .

Now,  $p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1 = -8 + 12 - 6 + 1 = -1 \neq 0$

Thus, by the Factor Theorem,  $g(x) = x + 2$  is not a factor of  $p(x) = x^3 + 3x^2 + 3x + 1$ .

(iii)  $p(x) = x^3 - 4x^2 + x + 6$ ,  $g(x) = x - 3$

Zero of  $g(x) = x - 3$  is 3 (obtained by putting  $g(x) = 0$ )

According to Factor Theorem, if  $g(x) = x - 3$  is a factor of  $p(x)$ , then  $p(3) = 0$ .

Now,  $p(3) = (3)^3 - 4(3)^2 + (3) + 6 = 27 - 36 + 3 + 6 = 0$

Thus, by the Factor Theorem,  $g(x) = x - 3$  is a factor of  $p(x) = x^3 - 4x^2 + x + 6$ .

3. Find the value of  $k$ , if  $x - 1$  is a factor of  $p(x)$  in each of the following cases:

(i)  $p(x) = x^2 + x + k$

(ii)  $p(x) = 2x^2 + kx + \sqrt{2}$

(iii)  $p(x) = kx^2 - \sqrt{2}x + 1$

(iv)  $p(x) = kx^2 - 3x + k$

**Ans** Zero of  $x - 1$  is 1 (obtained by putting  $x - 1 = 0$ ).

According to Factor Theorem, If  $x - 1$  is a factor of  $p(x)$ , then  $p(1) = 0$ .

(i)  $p(x) = x^2 + x + k$

»  $p(1) = (1)^2 + (1) + k = 0$  (From Factor Theorem)

»  **$k = -2$**

(ii)  $p(x) = 2x^2 + kx + \sqrt{2}$

»  $p(1) = 2(1)^2 + k(1) + \sqrt{2} = 0$  (From Factor Theorem)

»  **$k = -(2 + \sqrt{2})$**

(iii)  $p(x) = kx^2 - \sqrt{2}x + 1$

»  $p(1) = k(1)^2 - \sqrt{2}(1) + 1 = 0$  (From Factor Theorem)

»  **$k = \sqrt{2} - 1$**

$$(iv) p(x) = kx^2 - 3x + k$$

$$\gg p(1) = k(1)^2 - 3(1) + k = 0 \text{ (From Factor Theorem)}$$

$$\gg k = 3/2$$

4. Factorise :

$$(i) 12x^2 - 7x + 1$$

$$(ii) 2x^2 + 7x + 3$$

$$(iii) 6x^2 + 5x - 6$$

$$(iv) 3x^2 - x - 4$$

**Ans** (i)  $12x^2 - 7x + 1$

By splitting the middle term method,

$$p + q = -7 \text{ \& } pq = 12 \times 1 = 12$$

$$\text{So } p = -3 \text{ and } q = -4$$

$$\gg 12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$$

$$= 3x(4x - 1) - 1(4x - 1)$$

$$= (3x - 1)(4x - 1)$$

$$(ii) 2x^2 + 7x + 3$$

By splitting the middle term method,

$$p + q = 7 \text{ \& } pq = 2 \times 3 = 6$$

$$\text{So } p = 6 \text{ and } q = 1$$

$$\gg 2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$$

$$= 2x(x + 3) + 1(x + 3)$$

$$= (x + 3)(2x + 1)$$

$$(iii) 6x^2 + 5x - 6$$

By splitting the middle term method,

$$p + q = 5 \text{ \& } pq = 6 \times -6 = -36$$

$$\text{So } p = 9 \text{ and } q = -4$$

$$\gg 6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$$

$$= 3x(2x + 3) - 2(2x + 3)$$

$$= (2x + 3)(3x - 2)$$



(iv)  $3x^2 - x - 4$

By splitting the middle term method,

$p + q = -1$  &  $pq = 3 \times -4 = -12$

So  $p = -4$  and  $q = 3$

$\gg 3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

5. Factorise :

(i)  $x^3 - 2x^2 - x + 2$

(ii)  $x^3 - 3x^2 - 9x - 5$

(iii)  $x^3 + 13x^2 + 32x + 20$

(iv)  $2y^3 + y^2 - 2y - 1$

**Ans** (i) Let  $p(x) = x^3 - 2x^2 - x + 2$

All the factors of 2 must be considered. The factors are  $\pm 1$  &  $\pm 2$ .

By trial, we get  $p(1) = (1)^3 - 2(1)^2 - (1) + 2 = 0$ .

Thus,  $(x - 1)$  is a factor of  $p(x)$ .

Dividing  $p(x)$  by  $(x - 1)$ , we get

$$\begin{array}{r}
 \phantom{x-1} \overline{) \begin{array}{r} x^3 - 2x^2 - x + 2 \\ x^3 - x^2 \\ \hline -x^2 - x + 2 \\ -x^2 + x \\ \hline -2x + 2 \\ -2x + 2 \\ \hline 0 \end{array}} \\
 \phantom{x-1} \overline{) \begin{array}{r} x^3 - 2x^2 - x + 2 \\ x^3 - x^2 \\ \hline -x^2 - x + 2 \\ -x^2 + x \\ \hline -2x + 2 \\ -2x + 2 \\ \hline 0 \end{array}}
 \end{array}$$

Now, we know that

Dividend = Divisor X Quotient + Remainder

$\gg x^3 - 2x^2 - x + 2 = (x - 1)(x^2 - x - 2) + 0$

$x^2 - x - 2$  can be factorised further using splitting the middle term,

$$x^2 - x - 2 = x^2 - 2x + x - 2 = x(x - 2) + 1(x - 2) = (x - 2)(x + 1)$$

$$\text{Hence, } x^3 - 2x^2 - x + 2 = (x - 1)(x + 1)(x - 2)$$

(ii) Let  $p(x) = x^3 - 3x^2 - 9x - 5$

All the factors of 5 must be considered. The factors are  $\pm 1$  &  $\pm 5$ .

$$\text{By trial, we get } p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5 = -1 - 3 + 9 - 5 = 0.$$

Thus,  $(x + 1)$  is a factor of  $p(x)$ .

Dividing  $p(x)$  by  $(x + 1)$ , we get

$$\begin{array}{r}
 \phantom{x+1} \overline{x^2 - 4x - 5} \\
 x+1 \overline{) \phantom{x^2} x^3 - 3x^2 - 9x - 5} \\
 \underline{x^3 + x^2} \phantom{- 9x - 5} \\
 \phantom{x^3} -4x^2 - 9x - 5 \\
 \underline{-4x^2 - 4x} \phantom{- 5} \\
 \phantom{-4x^2} -5x - 5 \\
 \underline{-5x - 5} \\
 \phantom{-4x^2 - 9x} 0
 \end{array}$$

Now, we know that

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\gg x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5) + 0$$

$x^2 - 4x - 5$  can be factorised further using splitting the middle term,

$$x^2 - 4x - 5 = x^2 - 5x + x - 5 = x(x - 5) + 1(x - 5) = (x - 5)(x + 1)$$

$$\text{Hence, } x^3 - 3x^2 - 9x - 5 = (x + 1)(x + 1)(x - 5)$$

(iii) Let  $p(x) = x^3 + 13x^2 + 32x + 20$

All the factors of 20 must be considered. The factors are  $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10$  &  $\pm 20$ .

$$\text{By trial, we get } p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20 = -1 + 13 - 32 + 20 = 0.$$

Thus,  $(x + 1)$  is a factor of  $p(x)$ .

Dividing  $p(x)$  by  $(x + 1)$ , we get

$$\begin{array}{r}
 x^2 + 12x + 20 \\
 x + 1 \overline{) x^3 + 13x^2 + 32x + 20} \\
 \underline{x^3 + x^2} \phantom{+ 20} \\
 12x^2 + 32x + 20 \\
 \underline{12x^2 + 12x} \phantom{+ 20} \\
 20x + 20 \\
 \underline{20x + 20} \\
 0
 \end{array}$$

Now, we know that

Dividend = Divisor X Quotient + Remainder

$$\gg x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20) + 0$$

$x^2 + 12x + 20$  can be factorised further using splitting the middle term,

$$x^2 + 12x + 20 = x^2 + 10x + 2x + 20 = x(x + 10) + 2(x + 10) = (x + 10)(x + 2)$$

$$\text{Hence, } x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$$

(iv) Let  $p(y) = 2y^3 + y^2 - 2y - 1$

By trial, we get  $p(1) = 2(1)^3 + (1)^2 - 2(1) - 1 = 2 + 1 - 2 - 1 = 0$ .

Thus,  $(y - 1)$  is a factor of  $p(y)$ .

Dividing  $p(y)$  by  $(y - 1)$ , we get

$$\begin{array}{r}
 2y^2 + 3y + 1 \\
 y - 1 \overline{) 2y^3 + y^2 - 2y - 1} \\
 \underline{2y^3 - 2y^2} \phantom{- 1} \\
 3y^2 - 2y - 1 \\
 \underline{3y^2 - 3y} \phantom{- 1} \\
 y - 1 \\
 \underline{y - 1} \\
 0
 \end{array}$$

Now, we know that

Dividend = Divisor X Quotient + Remainder

$$\gg 2y^3 + y^2 - 2y - 1 = (y - 1)(2y^2 + 3y + 1) + 0$$

$2y^2 + 3y + 1$  can be factorised further using splitting the middle term,

$$2y^2 + 3y + 1 = 2y^2 + 2y + y + 1 = 2y(y + 1) + 1(y + 1) = (y + 1)(2y + 1)$$

$$\text{Hence, } 2y^3 + y^2 - 2y - 1 = (y - 1)(y + 1)(2y + 1)$$

### EXERCISE 2.5

1. Use suitable identities to find the following products:

(i)  $(x + 4)(x + 10)$

(ii)  $(x + 8)(x - 10)$

(iii)  $(3x + 4)(3x - 5)$

(iv)  $(y^2 + 3/2)(y^2 - 3/2)$

(v)  $(3 - 2x)(3 + 2x)$

**Ans** (i)  $(x + 4)(x + 10)$

Using identity,  $(x + a)(x + b) = x^2 + (a + b)x + ab$ , we have

$$(x + 4)(x + 10) = x^2 + (4 + 10)x + (4)(10) = x^2 + 14x + 40$$

(ii)  $(x + 8)(x - 10)$

Using identity,  $(x + a)(x + b) = x^2 + (a + b)x + ab$ , we have

$$(x + 8)(x - 10) = x^2 + (8 - 10)x + (8)(-10) = x^2 - 2x - 80$$

(iii)  $(3x + 4)(3x - 5)$

$$= 9(x + 4/3)(x - 5/3)$$

Using identity,  $(x + a)(x + b) = x^2 + (a + b)x + ab$ , we have

$$(3x + 4)(3x - 5) = 9(x + 4/3)(x - 5/3) = 9[x^2 + (4/3 - 5/3)x + (4/3)(-5/3)] = 9x^2 - 3x - 20$$

(iv)  $(y^2 + 3/2)(y^2 - 3/2)$

Using identity,  $(x - y)(x + y) = x^2 - y^2$ , we have

$$(y^2 + 3/2)(y^2 - 3/2) = (y^2)^2 - (3/2)^2 = y^4 - 9/4$$

(v)  $(3 - 2x)(3 + 2x)$

Using identity,  $(x - y)(x + y) = x^2 - y^2$ , we have

$$(3 - 2x)(3 + 2x) = (3)^2 - (2x)^2 = 9 - 4x^2$$

2. Evaluate the following products without multiplying directly:

(i)  $103 \times 107$

(ii)  $95 \times 96$

(iii)  $104 \times 96$

**Ans** (i)  $103 \times 107$

$$= (100 + 3)(100 + 7)$$

Using identity,  $(x + a)(x + b) = x^2 + (a + b)x + ab$ , we have

$$103 \times 107 = (100 + 3)(100 + 7) = (100)^2 + (3 + 7)100 + (3)(7)$$

$$= 10000 + 1000 + 21 = \mathbf{11021}$$

(ii)  $95 \times 96$

$$= (100 - 5)(100 - 4)$$

Using identity,  $(x + a)(x + b) = x^2 + (a + b)x + ab$ , we have

$$95 \times 96 = (100 - 5)(100 - 4) = (100)^2 + (-5 - 4)100 + (-5)(-4)$$

$$= 10000 - 900 + 20 = \mathbf{9120}$$

(iii)  $104 \times 96$

$= (100 + 4) (100 - 4)$

Using identity,  $(x + y) (x - y) = x^2 - y^2$ , we have

$104 \times 96 = (100 + 4) (100 - 4) = (100)^2 - (4)^2$

$= 10000 - 16 = \mathbf{9984}$

3. Factorise the following using appropriate identities:

(i)  $9x^2 + 6xy + y^2$       (ii)  $4y^2 - 4y + 1$       (iii)  $x^2 - y^2/100$

**Ans** (i)  $9x^2 + 6xy + y^2$

$= (3x)^2 + 2 (3x) (y) + (y)^2$

$= (3x + y)^2 = \mathbf{(3x + y) (3x + y)}$  [ using identity,  $x^2 + 2xy + y^2 = (x + y)^2$ ]

(ii)  $4y^2 - 4y + 1$

$= (2y)^2 - 2 (2y) (1) + (1)^2$

$= (2y - 1)^2 = \mathbf{(2y - 1) (2y - 1)}$  [ using identity,  $x^2 - 2xy + y^2 = (x - y)^2$ ]

(iii)  $x^2 - y^2/100$

$= (x)^2 - (y/10)^2$

$= \mathbf{(x + y/10) (x - y/10)}$  [ using identity,  $x^2 - y^2 = (x + y) (x - y)$ ]

4. Expand each of the following, using suitable identities:

(i)  $(x + 2y + 4z)^2$       (ii)  $(2x - y + z)^2$       (iii)  $(-2x + 3y + 2z)^2$

(iv)  $(3a - 7b - c)^2$       (v)  $(-2x + 5y - 3z)^2$       (vi)  $[a/4 - b/2 + 1]^2$

**Ans** (i)  $(x + 2y + 4z)^2$

Using identity,  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ , we have

$(x + 2y + 4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2 (x) (2y) + 2 (2y) (4z) + 2 (4z) (x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii)  $(2x - y + z)^2$

Using identity,  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ , we have

$$\begin{aligned}(2x - y + z)^2 &= (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x) \\ &= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz\end{aligned}$$

(iii)  $(-2x + 3y + 2z)^2$

Using identity,  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ , we have

$$\begin{aligned}(-2x + 3y + 2z)^2 &= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x) \\ &= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz\end{aligned}$$

(iv)  $(3a - 7b - c)^2$

Using identity,  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ , we have

$$\begin{aligned}(3a - 7b - c)^2 &= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a) \\ &= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac\end{aligned}$$

(v)  $(-2x + 5y - 3z)^2$

Using identity,  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ , we have

$$\begin{aligned}(-2x + 5y - 3z)^2 &= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x) \\ &= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz\end{aligned}$$

(vi)  $[a/4 - b/2 + 1]^2$

Using identity,  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ , we have

$$\begin{aligned}[a/4 - b/2 + 1]^2 &= (a/4)^2 + (-b/2)^2 + (1)^2 + 2(a/4)(-b/2) + 2(-b/2)(1) + 2(1)(a/4) \\ &= a^2/16 + b^2/4 + 1 - ab/4 - b + a/2\end{aligned}$$

5. Factorise:

(i)  $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii)  $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

**Ans** (i)  $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(2x)(-4z)$$

$$= (2x + 3y - 4z)^2 \text{ [ using identity } x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2 \text{ ]}$$

$$= (2x + 3y - 4z)(2x + 3y - 4z)$$

(ii)  $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)(y) + 2(y)(2\sqrt{2}z) + 2(-\sqrt{2}x)(2\sqrt{2}z)$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2 \text{ [ using identity } x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2 \text{ ]}$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$$

6. Write the following cubes in expanded form:

(i)  $(2x + 1)^3$

(ii)  $(2a - 3b)^3$

(iii)  $[3x/2 + 1]^3$

(iv)  $[x - 2y/3]^3$

**Ans** (i)  $(2x + 1)^3$

Using identity,  $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ , we have

$$(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1) = 8x^3 + 1 + 12x^2 + 6x$$

$$= 8x^3 + 12x^2 + 6x + 1$$

(ii)  $(2a - 3b)^3$

Using identity,  $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ , we have

$$(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$$

$$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$$

(iii)  $[3x/2 + 1]^3$

Using identity,  $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ , we have

$$[3x/2 + 1]^3 = (3x/2)^3 + (1)^3 + 3(3x/2)(1)(3x/2 + 1) = 27x^3/8 + 1 + 27x^2/4 + 9x/2$$

$$= 27x^3/8 + 27x^2/4 + 9x/2 + 1$$



(iv)  $[x - 2y/3]^3$

Using identity,  $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ , we have

$$[x - 2y/3]^3 = (x)^3 - (2y/3)^3 - 3(x)(2y/3)(x - 2y/3)$$

$$= x^3 - 8y^3/27 - 2x^2y + 4xy^2/3$$

7. Evaluate the following using suitable identities:

(i)  $(99)^3$                       (ii)  $(102)^3$                       (iii)  $(998)^3$

**Ans** (i)  $(99)^3$

$$= (100 - 1)^3$$

Using identity,  $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ , we have

$$(99)^3 = (100 - 1)^3 = (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$$

$$= 1000000 - 1 - 30000 + 300 = \mathbf{970299}$$

(ii)  $(102)^3$

$$= (100 + 2)^3$$

Using identity,  $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ , we have

$$(102)^3 = (100 + 2)^3 = (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$$

$$= 1000000 + 8 + 60000 + 1200 = \mathbf{1061208}$$

(iii)  $(998)^3$

$$= (1000 - 2)^3$$

Using identity,  $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ , we have

$$(998)^3 = (1000 - 2)^3 = (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$$

$$= 1000000000 - 8 - 6000000 + 12000 = \mathbf{994011992}$$

8. Factorise each of the following:

(i)  $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii)  $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii)  $27 - 125a^3 - 135a + 225a^2$

(iv)  $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v)  $27p^3 - 1/216 - 9p^2/2 + p/4$

**Ans** (i)  $8a^3 + b^3 + 12a^2b + 6ab^2$

$$= (2a)^3 + (b)^3 + 6ab(2a + b)$$

$$\begin{aligned}
 &= (2a)^3 + (b)^3 + 3(2a)(b)(2a+b) \\
 &= (2a+b)^3 \text{ [ Using identity, } x^3 + y^3 + 3xy(x+y) = (x+y)^3 \text{ ]} \\
 &= \mathbf{(2a+b)(2a+b)(2a+b)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } &8a^3 - b^3 - 12a^2b + 6ab^2 \\
 &= (2a)^3 - (b)^3 - 6ab(2a-b) \\
 &= (2a)^3 - (b)^3 - 3(2a)(b)(2a-b) \\
 &= (2a-b)^3 \text{ [ Using identity, } x^3 - y^3 - 3xy(x-y) = (x-y)^3 \text{ ]} \\
 &= \mathbf{(2a-b)(2a-b)(2a-b)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) } &27 - 125a^3 - 135a + 225a^2 \\
 &= (3)^3 - (5a)^3 - 45a(3-5a) \\
 &= (3)^3 - (5a)^3 - 3(3)(5a)(3-5a) \\
 &= (3-5a)^3 \text{ [ Using identity, } x^3 - y^3 - 3xy(x-y) = (x-y)^3 \text{ ]} \\
 &= \mathbf{(3-5a)(3-5a)(3-5a)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) } &64a^3 - 27b^3 - 144a^2b + 108ab^2 \\
 &= (4a)^3 - (3b)^3 - 36ab(4a-3b) \\
 &= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a-3b) \\
 &= (4a-3b)^3 \text{ [ Using identity, } x^3 - y^3 - 3xy(x-y) = (x-y)^3 \text{ ]} \\
 &= \mathbf{(4a-3b)(4a-3b)(4a-3b)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v) } &27p^3 - 1/216 - 9p^2/2 + p/4 \\
 &= (3p)^3 - (1/6)^3 - 3(3p)(1/6)(3p-1/6) \\
 &= (3p-1/6)^3 \text{ [ Using identity, } x^3 - y^3 - 3xy(x-y) = (x-y)^3 \text{ ]} \\
 &= \mathbf{(3p-1/6)(3p-1/6)(3p-1/6)}
 \end{aligned}$$

9. Verify : (i)  $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$       (ii)  $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

**Ans** (i)  $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

We all know that,  $x^3 + y^3 + 3xy(x+y) = (x+y)^3$

$$\gg x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

$$= (x + y) [(x + y)^2 - 3xy] = (x + y) (x^2 + y^2 + 2xy - 3xy)$$

$$= (x + y) (x^2 - xy + y^2)$$

Hence,  $x^3 + y^3 = (x + y) (x^2 - xy + y^2)$  is verified.

$$(ii) x^3 - y^3 = (x - y) (x^2 + xy + y^2)$$

We all know that,  $x^3 - y^3 - 3xy(x - y) = (x - y)^3$

$$\gg x^3 - y^3 = (x - y)^3 + 3xy(x - y)$$

$$= (x - y) [(x - y)^2 + 3xy] = (x - y) (x^2 + y^2 - 2xy + 3xy)$$

$$= (x - y) (x^2 + xy + y^2)$$

Hence,  $x^3 - y^3 = (x - y) (x^2 + xy + y^2)$  is verified.

10. Factorise each of the following:

$$(i) 27y^3 + 125z^3$$

$$(ii) 64m^3 - 343n^3$$

[Hint : See Question 9.]

**Ans** (i)  $27y^3 + 125z^3$

$$= (3y)^3 + (5z)^3$$

$$= (3y + 5z) [(3y)^2 - (3y)(5z) + (5z)^2] \text{ \{Using identity, } x^3 + y^3 = (x + y)(x^2 - xy + y^2)\}$$

$$= (3y + 5z) (9y^2 + 25z^2 - 15yz)$$

$$(ii) 64m^3 - 343n^3$$

$$= (4m)^3 - (7n)^3$$

$$= (4m - 7n) [(4m)^2 + (4m)(7n) + (7n)^2] \text{ \{ Using identity, } x^3 - y^3 = (x - y)(x^2 + xy + y^2)\}$$

$$= (4m - 7n) (16m^2 + 49n^2 + 28mn)$$

11. Factorise :  $27x^3 + y^3 + z^3 - 9xyz$

**Ans**  $27x^3 + y^3 + z^3 - 9xyz$

$$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$$

$$= (3x + y + z) [(3x)^2 + (y)^2 + (z)^2 - (3x)(y) - (y)(z) - (z)(3x)] \quad \{ \text{Using identity, } x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) \}$$

$$= (3x + y + z) (9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$$

12. Verify that  $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z) [(x - y)^2 + (y - z)^2 + (z - x)^2]$

**Ans** We know the identity,

$$\begin{aligned} x^3 + y^3 + z^3 - 3xyz &= (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx) \\ &= \frac{1}{2} [2(x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)] \\ &= \frac{1}{2} (x + y + z) (2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx) \\ &= \frac{1}{2} (x + y + z) [(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (x^2 + z^2 - 2zx)] \\ &= \frac{1}{2} (x + y + z) [(x - y)^2 + (y - z)^2 + (z - x)^2] \text{ (Hence verified)} \end{aligned}$$

13. If  $x + y + z = 0$ , show that  $x^3 + y^3 + z^3 = 3xyz$ .

**Ans** We know the identity,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)$$

Now, if  $x + y + z = 0$ , then

$$\begin{aligned} x^3 + y^3 + z^3 - 3xyz &= (0) (x^2 + y^2 + z^2 - xy - yz - zx) = 0 \\ \Rightarrow x^3 + y^3 + z^3 &= 3xyz \end{aligned}$$

14. Without actually calculating the cubes, find the value of each of the following:

(i)  $(-12)^3 + (7)^3 + (5)^3$

(ii)  $(28)^3 + (-15)^3 + (-13)^3$

**Ans** (i)  $(-12)^3 + (7)^3 + (5)^3$

Let  $x = -12$ ,  $y = 7$  and  $z = 5$

Then,  $x + y + z = -12 + 7 + 5 = 0$

As we know that, if  $x + y + z = 0$ , then

$$x^3 + y^3 + z^3 = 3xyz$$

$$\Rightarrow (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5) = -1260$$

(ii)  $(28)^3 + (-15)^3 + (-13)^3$

Let  $x = 28$ ,  $y = -15$  and  $z = -13$

Then,  $x + y + z = 28 - 15 - 13 = 0$

As we know that, if  $x + y + z = 0$ , then

$$x^3 + y^3 + z^3 = 3xyz$$

$$\gg (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13) = \mathbf{16380}$$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

**Area :  $25a^2 - 35a + 12$**

(i)

**Area :  $35y^2 + 13y - 12$**

(ii)

**Ans** (i) Area =  $25a^2 - 35a + 12$

Area = Length x Breadth

Thus length and breadth can be calculated by factorising the given polynomial.

Now we have, Area =  $25a^2 - 35a + 12$

By splitting the middle term method,

$$p + q = -35 \text{ \& } pq = 25 \times 12 = 300$$

So  $p = -20$  and  $q = -15$

$$\gg 25a^2 - 35a + 12 = 25a^2 - 20a - 15a + 12$$

$$= 5a(5a - 4) - 3(5a - 4)$$

$$= (5a - 3)(5a - 4)$$

Hence, Length =  $5a - 3$  and Breadth =  $5a - 4$ .

(ii) Area =  $35y^2 + 13y - 12$

Area = Length x Breadth

Thus length and breadth can be calculated by factorising the given polynomial.

Now we have, Area =  $35y^2 + 13y - 12$

By splitting the middle term method,

$$p + q = 13 \text{ \& } pq = 35 \times -12 = -420$$

$$\text{So } p = 28 \text{ and } q = -15$$

$$\gg 35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$$

$$= 7y(5y + 4) - 3(5y + 4)$$

$$= (7y - 3)(5y + 4)$$

Hence, Length =  $7y - 3$  and Breadth =  $5y + 4$ .

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

**Volume :  $3x^2 - 12x$**

(i)

**Volume :  $12ky^2 + 8ky - 20k$**

(ii)

**Ans** (i) Volume =  $3x^2 - 12x$

Volume of cuboid = Length x Breadth x Height

Thus length, breadth and height can be calculated by factorising the given polynomial.

Now we have,

$$\text{Volume} = 3x^2 - 12x$$

$$= 3x(x - 4)$$

Hence one of the solution is: Length = 3, Breadth = x and Height =  $x - 4$ .

(ii) Volume =  $12ky^2 + 8ky - 20k$

Volume of cuboid = Length x Breadth x Height

Thus length, breadth and height can be calculated by factorising the given polynomial.

Now we have,

$$\text{Volume} = 12ky^2 + 8ky - 20k$$

$$= 4k(3y^2 + 2y - 5)$$

By splitting the middle term method,

$$p + q = 2 \text{ \& } pq = 3 \times -5 = -15$$

$$\text{So } p = 5 \text{ and } q = -3$$



$$\gg 4k (3y^2 + 2y - 5) = 4k (3y^2 + 5y - 3y - 5)$$

$$= 4k [y(3y + 5) - 1(3y + 5)]$$

$$= \mathbf{4k (3y + 5) (y - 1)}$$

Hence one of the solution is: Length =  $4k$ , Breadth =  $3y + 5$  and Height =  $y - 1$ .