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An Enlightening Path of Knowledge

GATE-2015 (CIVIL ENGINEERING) Forenoon Session Paper with Solutions

- The Paper was held on 8th February, 2015 (Forenoon) and consists of 65 questions carrying 100 marks.
- General Aptitude section has 10 questions and carry a total of 15 marks. Q.1 – Q.5 carry 1 mark each, and questions Q.6 – Q.10 carry 2 marks each.
- Civil Engineering section has 55 questions and carry a total of 85 marks. Q.1 – Q.25 carry 1 mark each, and questions Q.26 – Q.55 carry 2 marks each.
- All efforts have been made to make this information as accurate as possible; Relevant/Authentic Text Books may be consulted for further information**.**
- Any discrepancy found may be brought to our notice by sending an e-mail to egyanbodh@gmail.com.
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Section: General Aptitude

1. Extreme focus on syllabus and studying for tests has become such a dominant concern of Indian students that they close their minds to anything _____ to the requirements of the exam.
   (A) related
   (B) extraneous
   (C) outside
   (D) useful

Ans (B) extraneous

2. Select the pair that best expresses a relationship similar to that expressed in the pair:
   Children : Pediatrician
   (A) Adult : Orthopaedist
   (B) Females : Gynaecologist
   (C) Kidney : Nephrologist
   (D) Skin : Dermatologist

Ans (B) Females : Gynaecologist

- A Pediatrician is a child specialist, similarly a Gynaecologist is a female specialist.

3. A function f(x) is linear and has a value of 29 at x = –2 and 39 at x = 3. Find its value at x = 5.
   (A) 59
   (B) 45
   (C) 43
   (D) 35

Ans (C) 43

- Let f(x) = mx + c

  Now, f(x) = 29 at x = -2

  And f(x) = 39 at x = 3

  » 29 = -2m + c

  39 = 3m + c

  Solving above two equations, we get

  m = 2 and c = 33

  So f(x) = 2x + 33

  Therefore, at x = 5, f(x) = 2 X 5 + 33 = **43**

4. The Tamil version of _____ John Abraham-starrer Madras Cafe _____ cleared by the Censor Board with no cuts last week, but the film's distributors _____ no takers among the exhibitors for a release in Tamil Nadu _____ this Friday

   (A) Mr., was, found, on

   (B) a, was, found, at

   (C) the ,was, found, on

   (D) a, being , find at

Ans (C) the ,was, found, on

5. If ROAD is written as URDG, then SWAN should be written as:

   (A) VXDQ

   (B) VZDQ

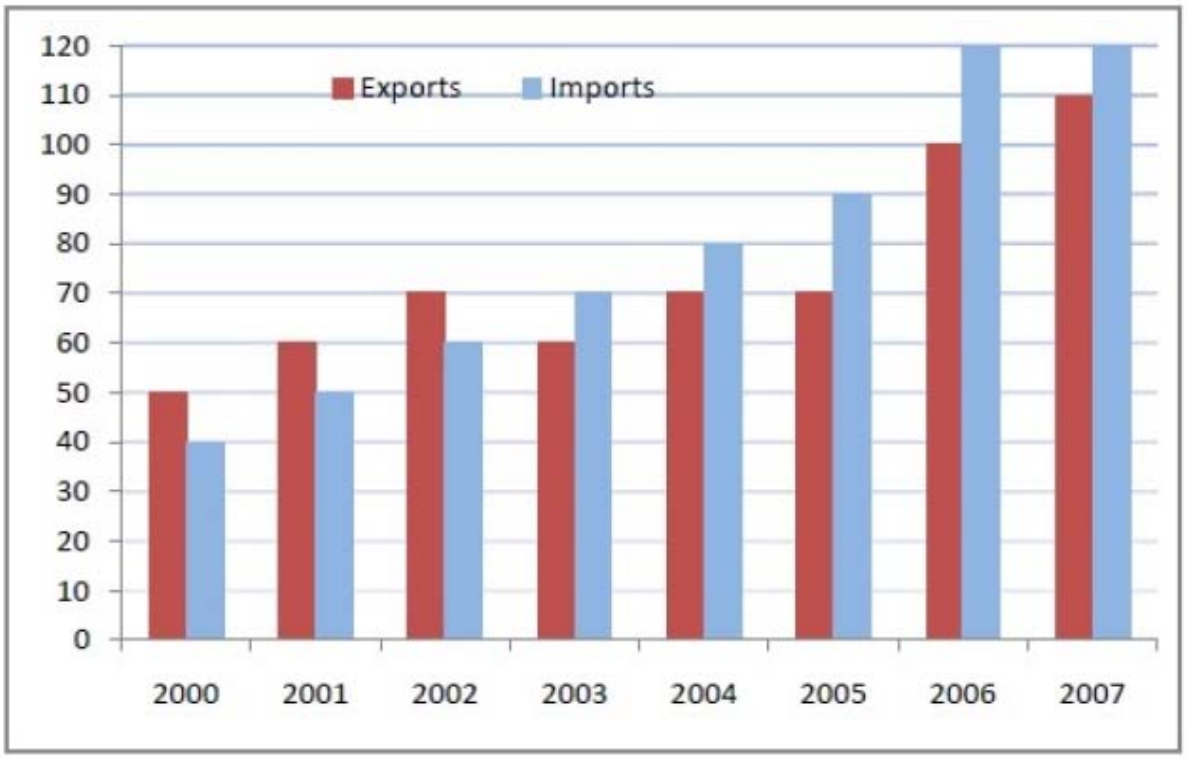
   (C) VZDP

   (D) UXDQ

Ans (B) VZDQ

- Clearly, each letter is replaced by the third following letter in the coded form as 'R' is replaced by 'U' and so on. Hence on changing each letter to third following letter, SWAN can be written as **'VZDQ'**

6. The exports and imports(in crores of Rs.) of a country from the year 2000 to 2007 are given in the following bar chart. In which year is the combined percentage increase in imports and exports the highest ?



Ans 2006

- Clearly, on seeing the bar chart, one can see that the import and export grows highest in 2006 (Jump of Rs 30 cr each).

  Import increase in 2006 = (120 – 90) / 90 = 33.33%

  Export increase in 2006 = (100 – 70) / 70 = 42.86%

  Therefore, combined % increase = 33.33 + 42.86 = **76.19%**

7. Most experts feel that in spite of possessing all the technical skills required to be a batsman of the highest order, he is unlikely to be so due to lack of requisite temperament. He was guilty of throwing away his wicket several times after working hard to lay a strong foundation. His critics pointed out that until he addressed this problem, success at the highest level will continue to elude him.

Which of the statement(s) below is/are logically valid and can be inferred from the above passage ?

(i) He was already a successful batsman at the highest level.

(ii) He has to improve his temperament in order to become a great batsman

(iii) He failed to make many of his good starts count.

(iv) Improving his technical skills will guarantee success.

(A) (iii) and (iv)

(B) (ii) and (iii)

(C) (i), (ii) and (iii)

(D) (ii) only

Ans (B) (ii) and (iii)

8. The head of a newly formed government desires to appoint five of the six selected members P,Q,R,S,T, and U to portfolios of Home, Power, Defense, Telecom, and Finance. U does not want any portfolio if S gets one of the five. R wants either Home or Finance or no portfolio. Q says that if S gets either Power or Telecom, then she must get the other one. T insists on a portfolio if P gets one.

Which is the valid distribution of portfolios ?

(A) P-Home, Q-Power, R-Defense, S-telecom, T-Finance

(B) R-Home, S-Power, P-Defense, Q-Telecom, T-Finance

(C) P-Home, Q-Power, T-Defense, S-Telecom, U-Finance

(D) Q-Home, U-Power, T-Defense, R-Telecom, P-Finance

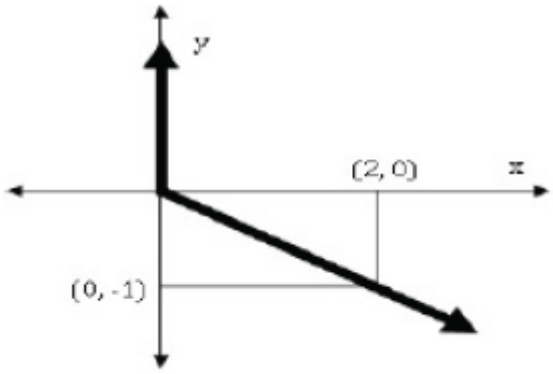
Ans (B) R-Home, S-Power, P-Defense, Q-Telecom, T-Finance

- Option (C) is not possible as U does not want any portfolio if S gets one of the five.

Options (A) & (D) are not possible as R wants either Home or Finance or no portfolio.

Hence option (B) is correct.

9. Choose the most appropriate equation for the function drawn as a thick line, in the plot below.



(A) x = y- |y|

(B) x = - (y- |y|)

(C) x = y + |y|

(D) x = - (y +|y|)

Ans (B) x = - (y- |y|)

- The value of 'x' is 2, when 'y' is -1. From the given options only option (B) satisfies this condition.

10. Alexander turned his attention towards India, since he had conquered Persia. Which one of the statements below is logically valid and can be inferred from the above sentence ?

(A) Alexander would not have turned his attention towards India had he not conquered Persia.

(B) Alexander was not ready to rest on his laurels, and wanted to march to India.

(C) Alexander was completely in control of his army and could command it to move towards India.

(D) Since Alexander's kingdom extended to Indian borders after the conquest of Persia, he was keen to move further.

Ans (A) Alexander would not have turned his attention towards India had he not conquered Persia.

Section: Civil Engineering

1. Workability of concrete can be measured using slump, compaction factor and Vebe time. Consider the following statements for workability of concrete:

   (i) As the slump increases, the Vebe time increases

   (ii) As the slump increases, the compaction factor increases

   Which of the following is TRUE ?

   (A) Both (i) and (ii) are True

   (B) Both (i) and (ii) are False

   (C) (i) is True and (ii) is False

   (D) (i) is False and (ii) is True

Ans (D) (i) is False and (ii) is True

- 1) Vebe time is defined as the time required for the shape of concrete to change from slump cone shape to cylindrical shape in seconds. Clearly, it decreases if the workability of concrete is more i.e. it has more water. **As the slump increases**, the workability increases and hence the **Vebe time decreases**.

  2) Compaction factor or degree of compaction is defined as,

  $$CF = \frac{\text{Weight of Partially Compacted Concrete}}{\text{Weight of Fully Compacted Concrete}}$$

  Clearly , CF is more when water is more i.e. it is directly proportional to the workability. Hence **as the slump increases, the compaction factor increases**.

2. Consider the following probability mass function (p.m.f) of a random variable X:

$$p(x,q) = \begin{cases} q & \text{if } X = 0 \\ 1 - q & \text{if } X = 1 \\ 0 & \text{otherwise} \end{cases}$$

   If q=0.4, the variance of X is _____.

Ans 0.24

- Variance of X is given by

Var (X) = $\sum pX^2 - (\sum pX)^2$

X	0	1	Otherwise (n)
P	q = 0.4	1 – q = 0.6	0

So, $\sum pX^2 = 0.4 \times 0^2 + 0.6 \times 1^2 + 0 \times n^2 = 0.6$

$\sum pX = 0.4 \times 0 + 0.6 \times 1 + 0 \times n = 0.6$

Hence, Var (X) = $0.6 - 0.6^2 = 0.6 - 0.36 =$ **0.24**

3. In a two-dimensional steady flow field, in a certain region of the x-y plane, the velocity component in the x-direction is given by $v_x = x^2$ and the density varies as $\rho = 1/x$. Which of the following is a valid expression for the velocity component in the y-direction, v_y ?

(A) $v_y = -x/y$

(B) $v_y = x/y$

(C) $v_y = -xy$

(D) $v_y = xy$

Ans (C) $v_y = -xy$

- For two-dimensional steady flow field,

$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$ (where, u and v are the velocity components in x and y directions respectively) ---------- (A)

If density 'ρ' is constant, the equation becomes

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Here, $u = v_x = x^2$, $v = v_y$ and $\rho = 1/x$

On putting above values in equation 'A', we get

$1 + \frac{1}{x}\frac{\partial v_y}{\partial y} = 0$

» $\frac{\partial v_y}{\partial y}$ = - x

On integrating both sides, we get

v_y = - xy

4. The two columns below show some parameters and their possible values.

Parameters	Value
P – Gross Command Area	I –100 hectares/cumec
Q – Permanent Wilting Point	II – 6° C
R – Duty of canal water	III – 1000 hectares
S – Delta of wheat	IV – 1000 cm
	V – 40 cm
	VI – 0.12

Which of the following options matches the parameters and the values correctly ?

(A) P-I, Q-II, R-III, S-IV

(B) P-III, Q-VI, R-I, S-V

(C) P-I, Q-V, R-VI, S-II

(D) P-III, Q-II, R-V, S-IV

Ans (B) P-III, Q-VI, R-I, S-V

- Clearly, the unit of gross command area is hectares, hence option III. The **permanent wilting point** is that water content at which plant can no longer extract sufficient water for its growth, and wilts up i.e. unitless and hence option VI. The unit of duty is hectares/cumec, hence option I. The unit of delta is cm, but the value given in option IV is absurd, hence option V. Thus answer is option 'B'.

5. Which of the following statements is TRUE for the relation between discharge velocity and seepage velocity ?

(A) Seepage velocity is always smaller than discharge velocity

(B) Seepage velocity can never be smaller than discharge velocity

(C) Seepage velocity is equal to the discharge velocity

(D) No relation between seepage velocity and discharge velocity can be established

Ans (B) Seepage velocity can never be smaller than discharge velocity

- Discharge velocity, v_d = Discharge/Cross-sectional Area = q/A (where A is area of solids and voids i.e. $A = A_s + A_v$)

  As the actual flow takes place only through voids, we have

  Seepage velocity, $v_s = q/A_v$

  Hence, $v_s > v_d$

  Further, $q = v_d A = v_s A_v$

  » $v_s = v_d (A/A_v)$

  $v_s = v_d (A \times L/A_v \times L)$ (where L is the length of specimen)

  » $v_s = v_d (V/V_v)$ (where V = total volume and V_V = volume of voids)

  Thus, $v_s = v_d / n$ (where n = porosity = V_v/V)

  Since $n < 1$, $v_s > v_d$

6. A fine-grained soil has 60% (by weight) silt content. The soil behaves as semi-solid when water content is between 15% and 28%. The soil behaves fluid-like when the water content is more than 40%. The 'Activity' of the soil is

   (A) 3.33

   (B) 0.42

   (C) 0.30

   (D) 0.20

Ans (C) 0.30

- Silt content by weight = 60%

  » Clay content or fraction by weight = 40%

  In the given question, Liquid limit (LL), w_L = 40%; Plastic limit (PL), w_P = 28%; Shrinkage limit (SL), w_s = 15%

  Now, Activity (A) = I_P / C (where I_P = Plasticity index and C = Clay fraction)

  $I_P = w_L - w_P = 40 - 28 = 12\%$

  Hence, A = 12/40 = 0.30

7. For what value of p the following set of equations will have no solution ?

$2x+3y = 5$

$3x+py = 10$

Ans 4.5

- For linear equations:

  $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ to have no solution, the condition is

  $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

  » $2/3 = 3/p$

  » p = 4.5

8. In a closed loop traverse of 1 km total length, the closing errors in departure and latitude are 0.3 m and 0.4 m, respectively. The relative precision of this traverse will be:

   (A) 1:5000

   (B) 1:4000

   (C) 1:3000

   (D) 1:2000

Ans (D) 1:2000

- Linear misclosure or error = $\sqrt{(\text{Departure misclosure})^2 + (\text{Latitude misclosure})^2}$

  $= \sqrt{0.3^2 + 0.4^2} = 0.5$

  » Relative precision = Linear misclosure/Traverse length = 0.5/1000 = **1:2000**

9. Total Kjeldahl Nitrogen (TKN) concentration (mg/L as N) in domestic sewage is the sum of the concentrations of :

   (A) Organic and inorganic nitrogen in sewage

   (B) Organic nitrogen and nitrate in sewage

   (C) Organic nitrogen and ammonia in sewage

   (D) Ammonia and nitrate in sewage

Ans (C) Organic nitrogen and ammonia in sewage

- Total Kjeldahl nitrogen (TKN) is a measure of the total organic and ammonia nitrogen in the wastewater. TKN gives a measure of the availability of nitrogen for building microbial cells, as well as the potential nitrogenous oxygen demand that will have to be satisfied.

10. A circular pipe has a diameter of 1m, bed slope of 1 in 1000, and Manning's roughness coefficient equal to 0.01. It may be treated as an open channel flow when it is flowing just full, i.e., the water level just touches the crest. The discharge in this condition is denoted by Q_{full}. Similarly, the discharge when the pipe is flowing half-full, i.e., with a flow depth of 0.5m, is denoted by Q_{half}. The ratio Q_{full}/Q_{half} is:

    (A) 1

    (B) $\sqrt{2}$

    (C) 2

    (D) 4

Ans (C) 2

- Discharge is given by, $Q = (1/n)\, A\, R^{2/3} S^{1/2}$

  For circular pipe of diameter 'd' flowing full,

  $A = \pi d^2/4$; Hydraulic radius, R = A/P (where P = wetted perimeter = πd)

  » $R = (\pi d^2/4) / \pi d = d/4$

  So, $Q_{full} = (1/n)\, (\pi d^2/4)\, (d/4)^{2/3} S^{1/2}$ ---------- (A)

  For circular pipe of diameter 'd' flowing half-full,

  $A = (1/2)(\pi d^2/4)$; Hydraulic radius, R = A/P (where P = wetted perimeter = $\pi d/2$)

  » $R = (1/2)(\pi d^2/4) / (\pi d/2) = d/4$

  So, $Q_{half} = (1/n)\, (1/2)(\pi d^2/4)\, (d/4)^{2/3} S^{1/2}$ ---------- (B)

  Hence, from equations (A) and (B),

  $Q_{full}/Q_{half} = 2$

11. Which of the following statements is FALSE ?

(A) Plumb line is along the direction of gravity

(B) Mean Sea Level (MSL) is used as a reference surface for establishing the horizontal control

(C) Mean Sea Level (MSL) is a simplification of the Geoid

(D) Geoid is an equi-potential surface of gravity

Ans (B)

- Mean Sea Level (MSL) is used as a reference surface for establishing the vertical control and not horizontal control.

12. Which of the following statements is NOT correct ?

(A) Loose sand exhibits contractive behavior upon shearing

(B) Dense sand when sheared under undrained condition, may lead to generation of negative pore pressure

(C) Black cotton soil exhibits expansive behavior

(D) Liquefaction is the phenomenon where cohesionless soil near the downstream side of dams or sheet-piles loses its shear strength due to high upward hydraulic gradient

Ans (D)

- **Liquefaction** is a state of saturated cohesionless soil when its shear strength is reduced to zero due to pore water pressure caused by vibration during an earthquake. The soil starts behaving like a liquid.

  It may be summarised that for liquefaction to occur, all the following five conditions must be satisfied:

  1. The soil is cohesionless.
  2. The soil is loose.
  3. The soil is saturated.
  4. There is shaking of ground of the required intensity and duration.
  5. The undrained conditions develop in the soil due to its limited permeability.

On the other hand, the phenomenon where cohesionless soil near the downstream side of dams or sheet-piles loses its shear strength due to high upward hydraulic gradient is called piping.

13. For steady incompressible flow through a closed-conduit of uniform cross-section, the direction of flow will always be :

(A) from higher to lower elevation

(B) from higher to lower pressure

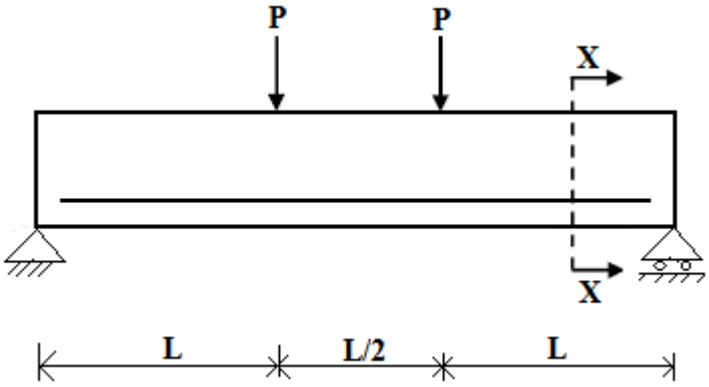
(C) from higher to lower velocity

(D) from higher to lower piezometric head

Ans (D) from higher to lower piezometric head

- **Piezometric head** = Pressure head + Datum head = $P/\rho g + Z$

14. Consider the singly reinforced beam shown in the figure below:



At cross-section XX, which of the following statements is TRUE at the limit state ?

(A) The variation of stress is linear and that of strain is non-linear

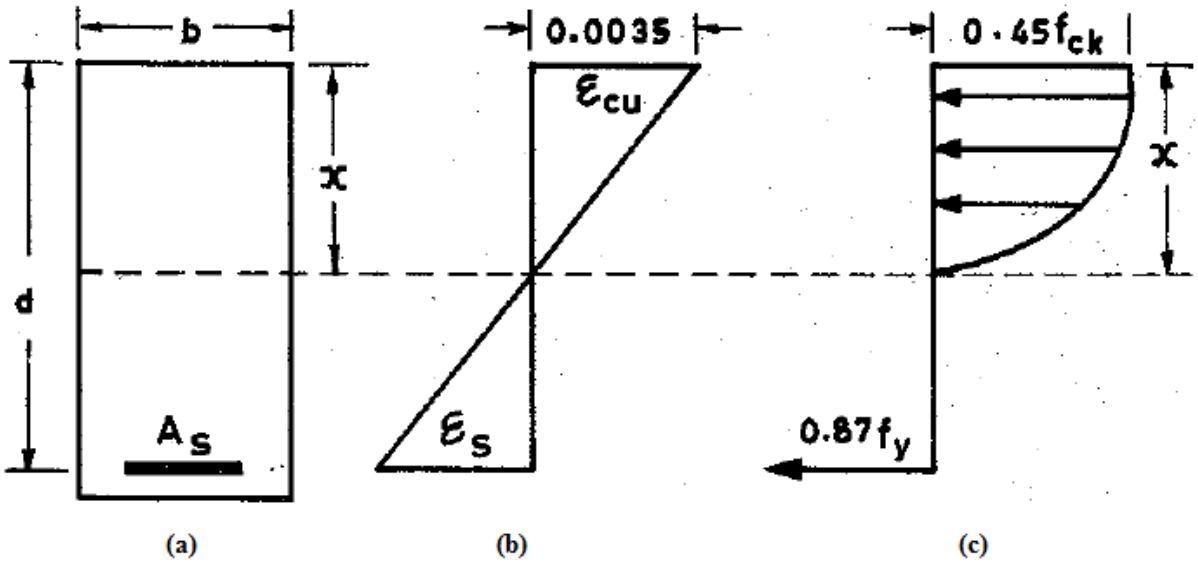
(B) The variation of strain is linear and that of stress is non – linear

(C) The variations of both stress and strain is linear

(D) The variation of both stress and strain is non-linear

Ans (B) The variation of strain is linear and that of stress is non – linear

- For limit state assumption is, Plane section remains plane i.e. strain variation is linear (proportional to distance from neutral axis)



(a) Section

(b) Strain diagram (plane sections remain plane)

(c) Stress diagram

15. The penetration value of a bitumen sample tested at 25°C is 80. When this sample is heated to 60°C and tested again, the needle of the penetration test apparatus penetrates the bitumen sample by 'd' mm. The value of 'd' CANNOT be less than _____ mm.

Ans 8 mm

- Penetration value of a bitumen sample tested at 25°C is 80 i.e. 8 mm.

  Variation of penetration with temperature is given by,

  log P = AT + K (where P = Penetration, A = Temperature susceptibility, T = Temperature, K = Constant)

  The value of 'A', generally varies from 0.015 to 0.06 for paving binders.

  From the question, $P_1 = 80$, $T_1 = 25$, $T_2 = 60$

  » Log P_1 = A (25) + K ----------(A)

  Log P_2 = A (60) + K ----------(B)

  Subtracting equation (A) from (B)

  Log(P_2/P_1) = 35 A

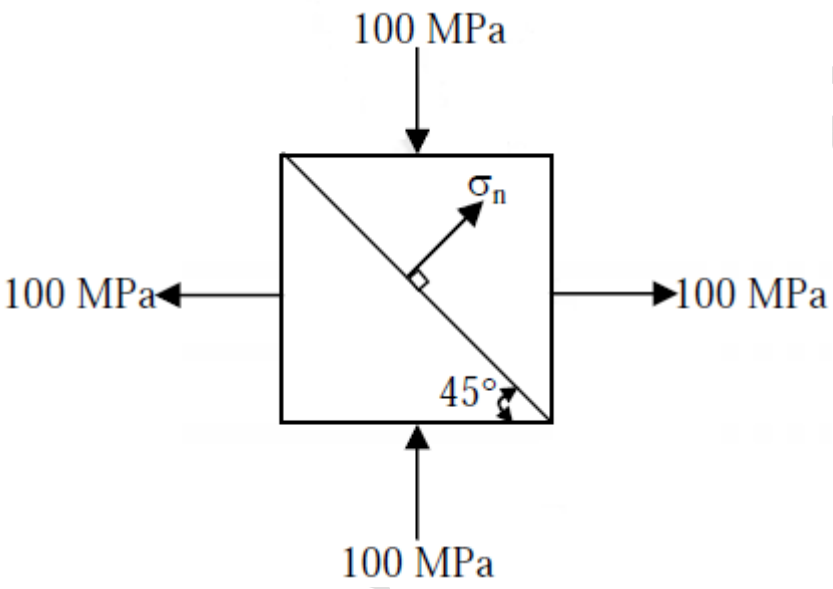
  For minimum value, lets take A = 0 i.e. bitumen is not at all temperature susceptible.

» $\text{Log}(P_2/P_1) = 0$

$P_2/P_1 = 10^0 = 1$

» $P_2 = P_1 = 80 = 8$ mm

16. Two Triangular wedges are glued together as shown in the following figure. The stress acting normal to the interface, σ_n is _____ MPa.



Ans 0

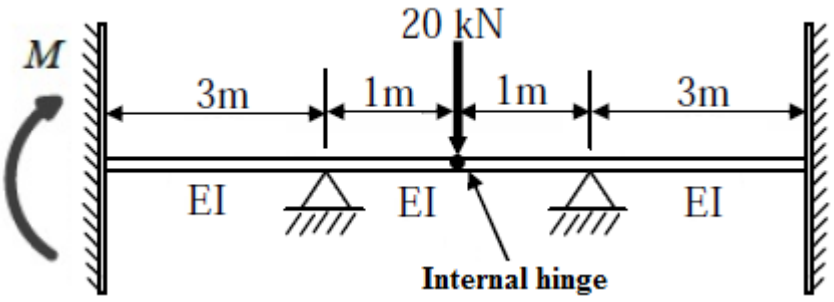
- $\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$

  Here, $\sigma_x = 100$ MPa (tensile); $\sigma_y = -100$ MPa (Compressive); $\theta = 45^0$

  On putting above values, we get

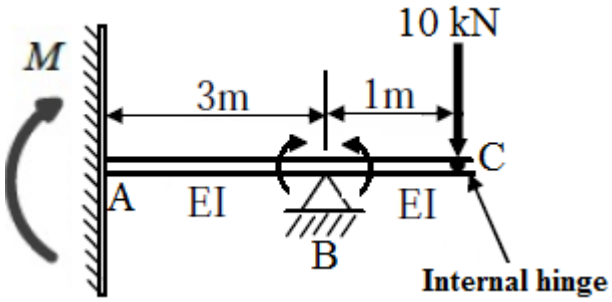
  $\sigma_n = (\sigma_x + \sigma_y)/2 = (100 - 100)/2 = 0$

17. For the beam shown below, the value of the support moment M is _____ kN-m.



Ans 5

- Splitting the beam at internal hinge and as the beam is symmetrical we get,



M_{BC} + 10 x 1 = 0

» M_{BC} = -10 kNm (anticlockwise)

Now at B, $M_{BA} + M_{BC} = 0$

» $M_{BA} = - M_{BC}$ = 10 kNm (clockwise)

$M_{AB} = (2EI/3)(2\theta_A + \theta_B) + M_{FAB}$

$M_{BA} = (2EI/3)(2\theta_B + \theta_A) + M_{FBA}$

In this case, $M_{FAB} = M_{FBA} = 0$; $\theta_A = 0$

» M = M_{AB} = M_{BA}/2 = 10/2 = 5 kNm (clockwise)

18. Which of the following statements CANNOT be used to describe free flow speed (u_f) of a traffic stream ?

(A) u_f is the speed when flow is negligible

(B) u_f is the speed when density is negligible

(C) u_f is affected by geometry and surface conditions of the road

(D) u_f is the speed at which flow is maximum and density is optimum

Ans (D) u_f is the speed at which flow is maximum and density is optimum.

- When the flow is very low, there is little interaction between individual vehicles. Drivers are therefore free to travel at the maximum possible speed. The absolute maximum speed is obtained when the **flow as well as density tends to 0 (**When the density on the highway is 0, the flow is also 0 because there are no vehicles on the highway), and it is known as the free flow speed (u_f). Hence option (A) and (B) are

correct. Further the magnitude of the free flow speed depends on the physical characteristics of the highway. Hence option (C) is correct.

19. The development length of a deformed reinforcement bar can be expressed as (1/k) ($\varphi\sigma_s/\tau_{bd}$) From the IS:456-2000, the value of k can be calculated as _____.

Ans 6.4

- From IS:456-2000, clause 26.2.1,

  Development length (L_d) = $\varphi\sigma_s/4\tau_{bd}$

  Further as per clause 26.2.1.1, for deformed reinforcement bars, the value of τ_{bd} is increased by 60%.

  Hence k = 4 X 1.6 = **6.4**

20. In an unconsolidated undrained triaxial test, it is observed that an increase in cell pressure from 150 kPa to 250 kPa leads to a pore pressure increase of 80 kPa. It is further observed that, an increase of 50 kPa in deviatoric stress results in an increase of 25 kPa in the pore pressure. The value of Skemptons's pore pressure parameter B is:

    (A) 0.5

    (B) 0.625

    (C) 0.8

    (D) 1.0

Ans (C) 0.8

- Skemptons's pore pressure parameter,

  B = Change in pore water pressure/Change in cell pressure = $\Delta u_3 / \Delta\sigma_3$

  From the question, $\Delta\sigma_3$ = 250 – 150 = 100 kPa; Δu_3 = 80 kPa

  » B = $\Delta u_3 / \Delta\sigma_3$ = 80/100 = 0.8

21. Solid waste generated from an industry contains only two components, X and Y as shown in the table below

Component	Composition (% weight)	Density (kg/m^3)
X	c_1	ρ_1
Y	c_2	ρ_2

Assuming $(c_1+c_2) = 100$, the composite density of the solid waste (ρ) is given by:

(A) $\frac{100}{(\frac{c_1}{\rho_1}+\frac{c_2}{\rho_2})}$

(B) $100\ (\frac{\rho_1}{c_1}+\frac{\rho_2}{c_2})$

(C) $100\ (c_1\rho_1 + c_2\rho_2)$

(D) $100\ (\frac{\rho_1\rho_2}{c_1\rho_1+ c_2\rho_2})$

Ans (A) $\frac{100}{(\frac{c_1}{\rho_1}+\frac{c_2}{\rho_2})}$

- Composite density of the solid waste (ρ) = Total weight / Total volume

  Volume of component X = c_1/ρ_1

  Volume of component Y = c_2/ρ_2

  Hence, Composite density = $\frac{c_1+ c_2}{(\frac{c_1}{\rho_1}+\frac{c_2}{\rho_2})} = \frac{\mathbf{100}}{(\frac{\mathbf{c_1}}{\mathbf{\rho_1}}+\frac{\mathbf{c_2}}{\mathbf{\rho_2}})}$ (because $c_1 + c_2 = 100$)

22. The integral $\int_{x_1}^{x_2} x^2 dx$ with $x_2 > x_1 > 0$ is evaluated analytically as well as numerically using a single application of the trapezoidal rule. If 'I' is the exact value of the integral obtained analytically and 'J' is the approximate value obtained using the trapezoidal rule, which of the following statements is correct about their relationship ?

(A) J > I

(B) J < I

(C) J = I

(D) Insufficient data to determine the relationship

Ans (A) J > I

- $I = \int_{x_1}^{x_2} x^2 dx = [\frac{x^3}{3}]_{x_1}^{x_2} = (x_2^3 - x_1^3) / 3 = (x_2 - x_1) (x_1^2 + x_2^2 + x_1x_2) / 3$

For trapezoidal rule,

$\int_a^b f(X)dx = (h/2) [(y_0 + y_n) + 2 (y_1 + y_2 + \text{----} + y_{n-1})]$ and $h = (b - a) / n$ (where y_0 and y_n are the values of f(x) when x = a and b respectively)

From the question, n = 1; a = x_1; b = x_2; $y_a = x_1^2$; $y_b = x_2^2$

» $h = x_2 - x_1$

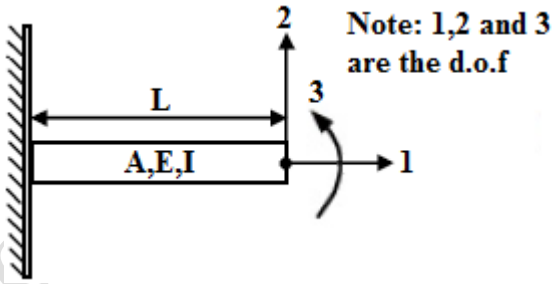
So, $J = \int_{x_1}^{x_2} x^2 dx = (x_2 - x_1)/2 [x_1^2 + x_2^2]$

» $J - I = (x_2 - x_1) [(x_1^2 + x_2^2) / 2 - (x_1^2 + x_2^2 + x_1x_2) / 3]$

$= (x_2 - x_1) (x_1^2 + x_2^2 - 2x_1x_2) / 6 = (x_2 - x_1) (x_2 - x_1)^2 / 6 = (x_2 - x_1)^3/6$

Hence J > I (because $x_2 > x_1$)

23. For the beam shown below, the stiffness coefficient K_{22} can be written as



(A) $6EI/L^2$

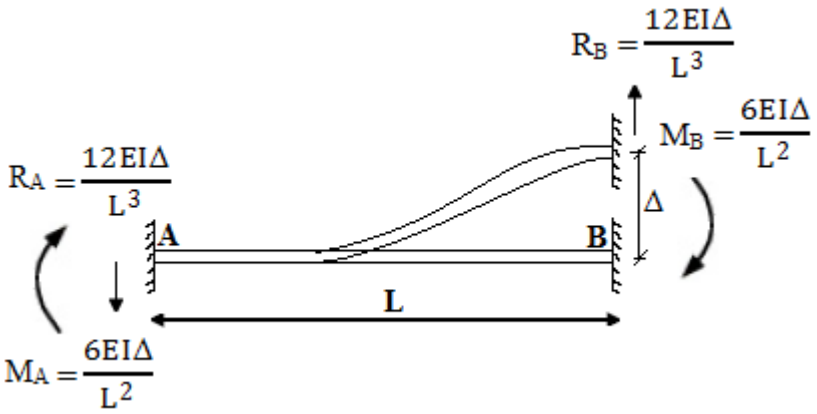
(B) $12EI/L^3$

(C) $3EI/L$

(D) $EI/6L^2$

Ans (B) $12EI/L^3$

- When displacement Δ is given in the direction of 2 while other possible displacements are prevented, the resulting shear forces and bending moments created are shown below.



K_{ij} denotes the force in direction i, due to a unit deformation (displacement) in direction j. Thus K_{22} is the force in direction 2, due to a unit deformation (displacement) in direction 2.

For determining K_{22}, we need to put $\Delta = 1$

Hence, $K_{22} = R_B =$ **$12EI/L^3$**

24. Which of the following statements is TRUE for degree of disturbance of collected soil sample?

(A) Thinner the sampler wall, lower the degree of disturbance of collected soil sample

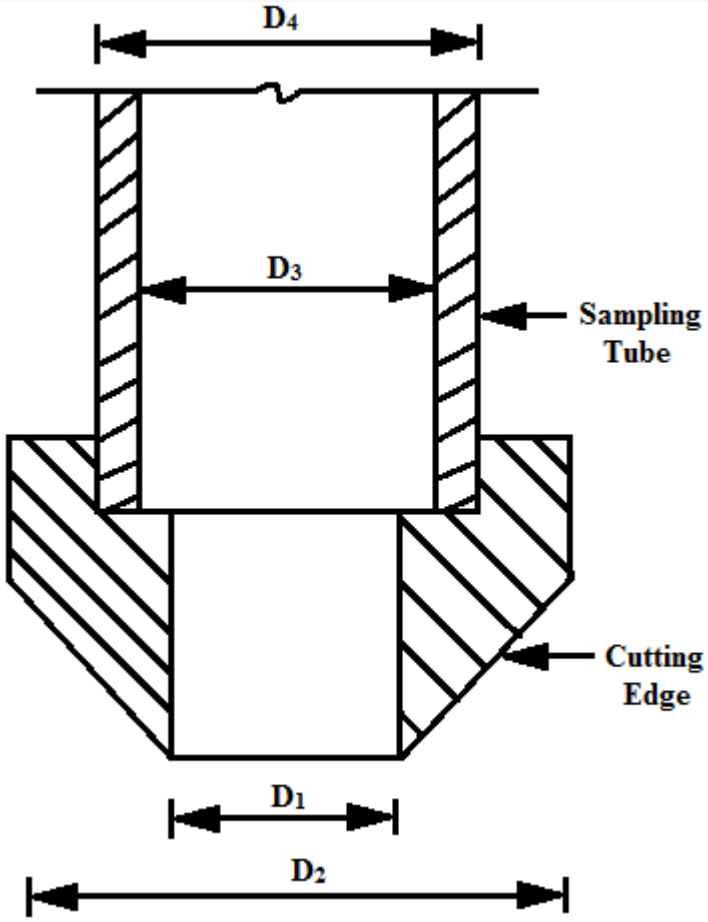
(B) Thicker the sampler wall, lower the degree of disturbance of collected soil sample

(C) Thickness of the sampler wall and the degree of disturbance of collected soil sample are unrelated

(D) The degree of disturbance of collected soil sample is proportional to the inner diameter of the sampling tube.

Ans (A) Thinner the sampler wall, lower the degree of disturbance of collected soil sample

- The diagram of the sampler is shown below:



The disturbance of the soil depends on area ratio. The smaller is the area ratio, less is the disturbance.

Area ratio = $\frac{D_2^2 - D_1^2}{D_1^2}$ x 100 ; for obtaining good quality undisturbed samples, the area ratio should be 10% or less.

For thin wall samplers, Area ratio < 10%

For thick wall samplers, Area ratio > 10%

25. Consider the following statements for air-entrained concrete:

(i) Air-entrainment reduces the water demand for a given level of workability

(ii) Use of air-entrained concrete is required in environments where cyclic freezing and thawing is expected

Which of the following is TRUE ?

(A) Both (i) and (ii) are True

(B) Both (i) and (ii) are False

(C) (i) is True and (ii) is False

(D) (i) is False and (ii) is True

Ans (A) Both (i) and (ii) are True

- Air entrained concrete is made by mixing a small quantity of air entraining agent or by using air entraining cement. These air entraining agents incorporate millions of non coalescing air bubbles, which will act as flexible ball bearings and enhance the workability.

  Further, these air pockets relieve internal pressure on the concrete by providing tiny chambers for water to expand into when it freezes (volume of ice is about 10% higher than the corresponding volume of water).

  The amount of entrained air is usually between four to seven percent of the volume of the concrete.

26. The 4-hr unit hydrograph for a catchment is given in the table below. What would be the maximum ordinate of the S-curve (in m^3/s) derived from this hydrograph ?

Time (hr)	0	2	4	6	8	10	12	14	16	18	20	22	24
Unit hydrograph ordinate (m^3/s)	0	0.6	3.1	10	13	9	5	2	0.7	0.3	0.2	0.1	0

Ans 22

- S-curve coordinates are calculated in the table given on next page:

Time (hr) (1)	Unit hydrograph ordinate (m^3/s) (2)	S-curve additions (cumec) (3)					S-curve ordinates (cumec) (2) + (3)
0	0						0
2	0.6						0.6
4	3.1	0					3.1
6	10	0.6					10.6
8	13	3.1	0				16.1
10	9	10	0.6				19.6
12	5	13	3.1	0			21.1
14	2	9	10	0.6			21.6
16	0.7	5	13	3.1	0		21.8
18	0.3	2	9	10	0.6		21.9
20	0.2	0.7	5	13	3.1	0	**22**
22	0.1	0.3	2	9	10	0.6	**22**
24	0	0.2	0.7	5	13	3.1	**22**

27. The composition of an air-entrained concrete is given below:

Water : 184 kg/m^3

Ordinary Portland Cement (OPC) : 368 kg/m^3

Sand : 606 kg/m^3

Coarse aggregate : 1155 kg/m^3

Assume the specific gravity of OPC, sand and coarse aggregate to be 3.14, 2.67 and 2.74, respectively. The air content is _____ $litres/m^3$.

Ans 50.3

- The question details can be shown in the table below:

Item number	Ingredients	Weight (kg/m^3)	Absolute volume (m^3)
1.	OPC	368	368/(3.14 x 1000) = 0.1172
2.	Water	184	184/1000 = 0.184
3.	Coarse Aggregate	1155	1155/(2.74 x 1000) = 0.4215
4.	Sand	606	606/(2.67 x 1000) = 0.227
5.	Air		V_A

Total absolute volume = 1 m^3

$V_{OPC} + V_W + V_{CA} + V_S + V_A = 1$

$0.1172 + 0.184 + 0.4215 + 0.227 + V_A = 1$

» V_A = 0.0503 = 0.0503 x 10^3 litres/m^3 = 50.3 litres/m^3

28. In a catchment, there are four rain – gauge stations, P,Q,R, and S. Normal annual precipitation values at these stations are 780 mm, 850 mm, 920 mm, and 980 mm, respectively. In the year 2013, stations Q,R, and S, were operative but P was not. Using the normal ratio method, the precipitation at station P for the year 2013 has been estimated as 860 mm. If the observed precipitation at stations Q and R for the year 2013 were 930 mm and 1010 mm, respectively, what was the observed precipitation (in mm) at station S for that year ?

Ans 1093.43

- Using normal ratio method and equating ratios of observed precipitation to normal annual precipitation we get precipitation at station P (P_p) as

$$\frac{P_p}{780} = \frac{930}{850} = \frac{1010}{920} = \frac{P_s}{980}$$

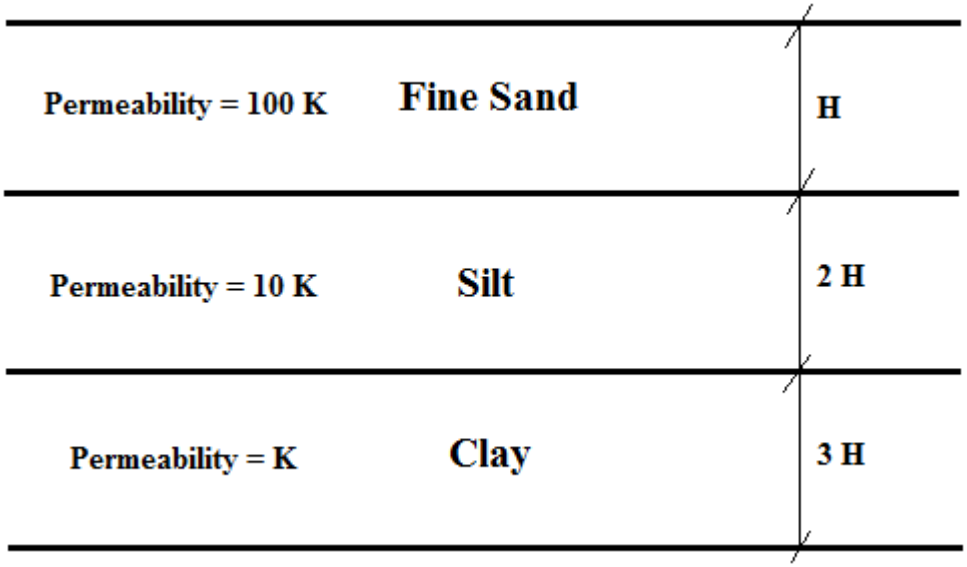
» $P_p = (1/3)\left[\frac{930}{850} \text{ X } 780 + \frac{1010}{920} \text{ X } 780 + \frac{P_s}{980} \text{ X } 780\right] = 860$

Hence, P_s = 1093.43

29. A non-homogeneous soil deposit consists of a silt layer sandwiched between a fine-sand layer at top and a clay layer below. Permeability of the silt layer is 10 times the permeability of the clay layer and one-tenth of the permeability of the sand layer. Thickness of the silt layer is 2 times the thickness of the sand layer and two-third of the thickness of the clay layer. The ratio of equivalent horizontal and equivalent vertical permeability of the deposit is _____.

Ans 10.96

- From the given data, the condition will be:



K_h = (100 K x H + 10 K x 2 H + K x 3 H) / (H + 2 H + 3 H)

» K_h = 20.5 K

$K_v = (H + 2H + 3H) / \left(\frac{H}{100K} + \frac{2H}{10K} + \frac{3H}{K}\right)$

» K_v = 1.87 K

Hence, K_h / K_v = 20.5 / 1.87 = 10.96

30. Consider the following differential equation:

x (ydx + xdy) cos(y/x) = y (xdy – ydx) sin(y/x)

Which of the following is the solution of the above equation (c is an arbitrary constant)?

(A) $\frac{x}{y}\cos\frac{y}{x} = C$

(B) $\frac{x}{y}\sin\frac{y}{x} = C$

(C) $xy\cos\frac{y}{x} = C$

(D) $xy\sin\frac{y}{x} = C$

Ans (C) $xy\cos\frac{y}{x} = C$

- Let y/x = v, then $\frac{dy}{dx} = v + x\frac{dv}{dx}$ and the equation becomes

x^2 [(y/x)dx + dy] cos(y/x) = yx [dy – (y/x)dx] sin(y/x)

$(v + \frac{dy}{dx})\cos v = v(\frac{dy}{dx} - v)\sin v$

$(2v + x\frac{dv}{dx})\cos v = v\,x\frac{dv}{dx}\sin v$

$x\frac{dv}{dx}(v\sin v - \cos v) = 2v\cos v$

(2/x) dx = (tanv – 1/v) dv

On integration we get

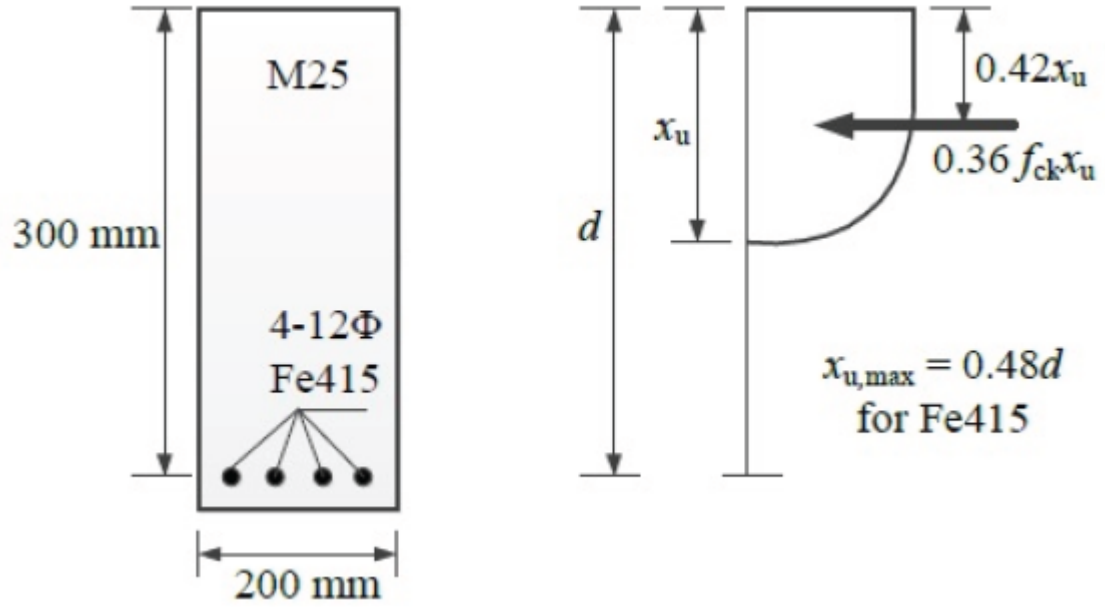
2 log x = log secv – log v + log C

$\log x^2 = \log(C\sec v/v)$

$vx^2 = C\sec v$

xy cos(y/x) = C (because v = y/x)

31. Consider the singly reinforced beam section given below(left figure). The stress block parameters for the cross - section from IS:456-2000 are also given below (right figure). The moment of resistance for the given section by the limit state method is _____ kN-m.



Ans 42.76 kNm

- Depth of neutral axis can be calculated as

  $x_u = (0.87\ f_y\ A_{st}) / (0.36\ f_{ck}\ b)$

  From the question, b = 200 mm, f_{ck} = 25 N/mm^2; f_y = 415 N/mm^2;

  $A_{st} = 4 \text{ X } \pi\ (12)^2/4 = 144\ \pi$ mm^2

  » x_u = 90.696 mm

  Hence, $M_u = 0.36\ f_{ck}\ b\ x_u\ (d - 0.42\ x_u)$

  » M_u = 0.36 X 25 X 200 X 90.696 (300 – 0.42 X 90.696) = 42.76 kNm

32. The concentration of Sulphur Dioxide (SO_2) in ambient atmosphere was measured as 30 µg/m^3. Under the same conditions, the above SO_2 concentration expressed in ppm is _____.

  Given: P/(RT) = 41.6 mol/m^3; where, P = Pressure; T = Temperature; R= universal gas constant; Molecular Weight of SO_2 = 64.

Ans 0.0112

- From the universal gas law, we have

  PV = nRT

  For n = 1 mole, V = RT/P

  From the question we have, P/RT = 41.6 mol/m^3

  » Volume of SO_2/mole = V = 1/41.6 = 24 X 10^{-3} m^3/mole

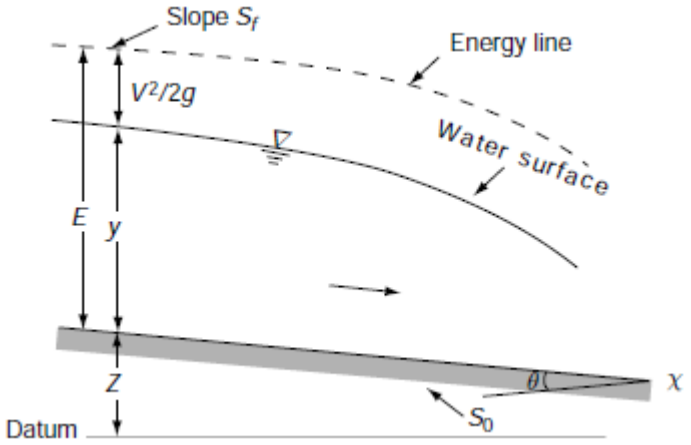
  Now, Conc. in ppm = (Conc. in μg/m^3) (Volume of gas/mole) / Molecular weight

  » Conc. of SO_2 in ppm = 30 X 24 X 10^{-3} / 64 = 0.0112

33. A short reach of a 2 m wide rectangular open channel has its bed level rising in the direction of flow at a slope of 1 in 10000. It carries a discharge of 4 m^3/s and its Manning's roughness coefficient is 0.01. The flow in this reach is gradually varying. At a certain section in this reach, the depth of flow was measured as 0.5 m. The rate of change of the water depth with distance, dy/dx, at this section is _____(use g = 10 m/s^2)

Ans 0.0032

- The schematic diagram of a gradually varied flow is



Total Energy, H = Z + E (where E = Specific Energy)

» H = Z + y + $V^2/2g$

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right)$$ ---------- (A)

Now, $\frac{dH}{dx}$ = - S_f = Energy slope; $\frac{dZ}{dx}$ = - S_0 = Channel bed slope

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{d}{dx}\left(\frac{q^2/y^2}{2g}\right) = -\left(\frac{q^2}{gy^3}\right)\frac{dy}{dx} = -\left(\frac{V^2}{gy}\right)\frac{dy}{dx}$$

Putting all the above values in equation (A), we get

$$-S_f = -S_0 + \frac{dy}{dx} + -\left(\frac{V^2}{gy}\right)\frac{dy}{dx}$$

» $\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$ (where F_r = Froud's number = $V/\sqrt{gy}$)

In the given question, S_0 = 1/10000 = - 0.0001; V = Q/A = 4/(2 x 0.5) = 4 m/s

» $F_r = 4/\sqrt{10 \times 0.5} = 1.789$

Now, $Q = (1/n)\, A\, R^{2/3} (S_f)^{1/2}$

$4 = (1/0.01)\,(2 \times 0.5)\,(2 \times 0.5 / 2 + 2 \times 0.5)^{2/3}\,(S_f)^{1/2}$

» $S_f = 0.0069$

Hence, $\mathbf{\frac{dy}{dx} = \frac{-0.0001 - 0.0069}{1 - 1.789^2} = 0.0032}$

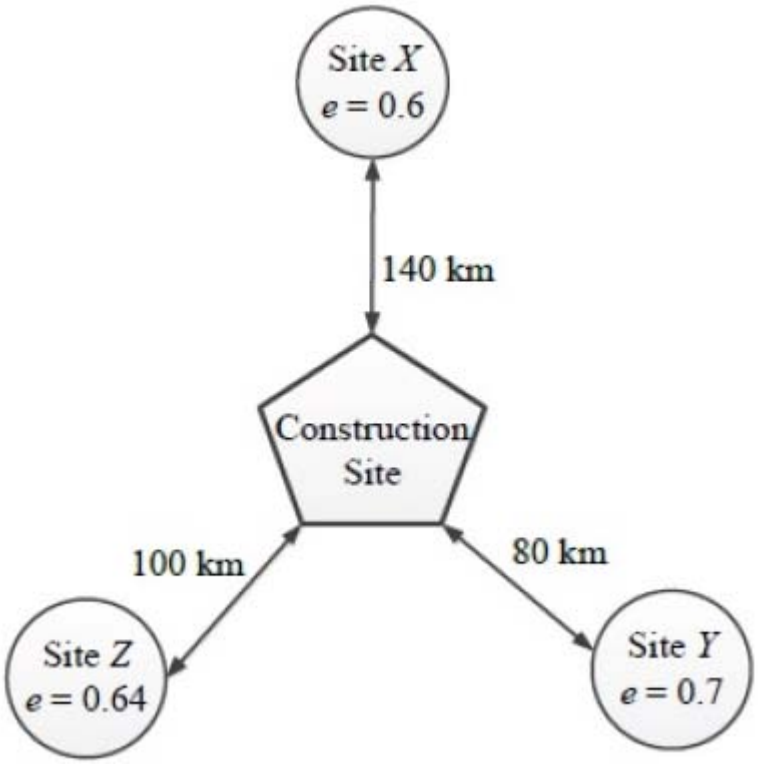
34. An earth embankment is to be constructed with compacted cohesionless soil. The volume of the embankment is 5000 m^3 and the target dry unit weight is 16.2 kN/m^3. Three nearby sites (see figure below) have been identified from where the required soil can be transported to the constructed site. The void ratios (e) of different sites are shown in the figure. Assume the specific gravity of soil to be 2.7 for all three sites. If the cost of transportation per km is twice the cost of excavation per m^3 of borrow pits, which site would you choose as the most economic solution? (use unit weight of water = 10 kN/m^3)

(A) Site X

(B) Site Y

(C) Site Z

(D) Any of the sites



Ans (A) Site X

- Weight of solids, $W_s = \gamma_d V = 16.2 \times 5000 = 81000$ kN/m^3

  » Volume of solids, $V_s = W_s/G\gamma_w = 81000 / (\quad 2.7 \times 10) = 3000$ m^3

  Thus, volume of solids required from all sites is 3000 m^3.

  Volume of soil required from site X, $V_X = V_v + V_s$

  » $V_X = eV_s + V_s = (1 + e)V_s = (1 + 0.6)\ 3000 = 4800$ m^3 (because $e = V_v/V_s$)

  Similarly, $V_Y = (1 + 0.7)\ 3000 = 5100$ m^3

  $V_Z = (1 + 0.64)\ 3000 = 4920$ m^3

  Let cost of excavation/m^3 = A

  » Cost of transportation/km = 2A

  Total cost for site X = 4800 A + 140 (2A) = 5080 A

  Total cost for site Y = 5100 A + 80 (2A) = 5260 A

  Total cost for site Z = 4920 A + 100 (2A) = 5120 A

  Hence, site X is most economical solution.

35. In a region with magnetic declination of 2°E, the magnetic Fore bearing(FB) of a line AB was measured as N79°50′E. There was local attraction at A. To determine the correct magnetic bearing of the line, a point O was selected at which there was no local attraction. The magnetic FB of line AO and OA were observed to be S52°40′E and N50°20′W, respectively. What is the true FB of line AB ?

(A) N81°50′E

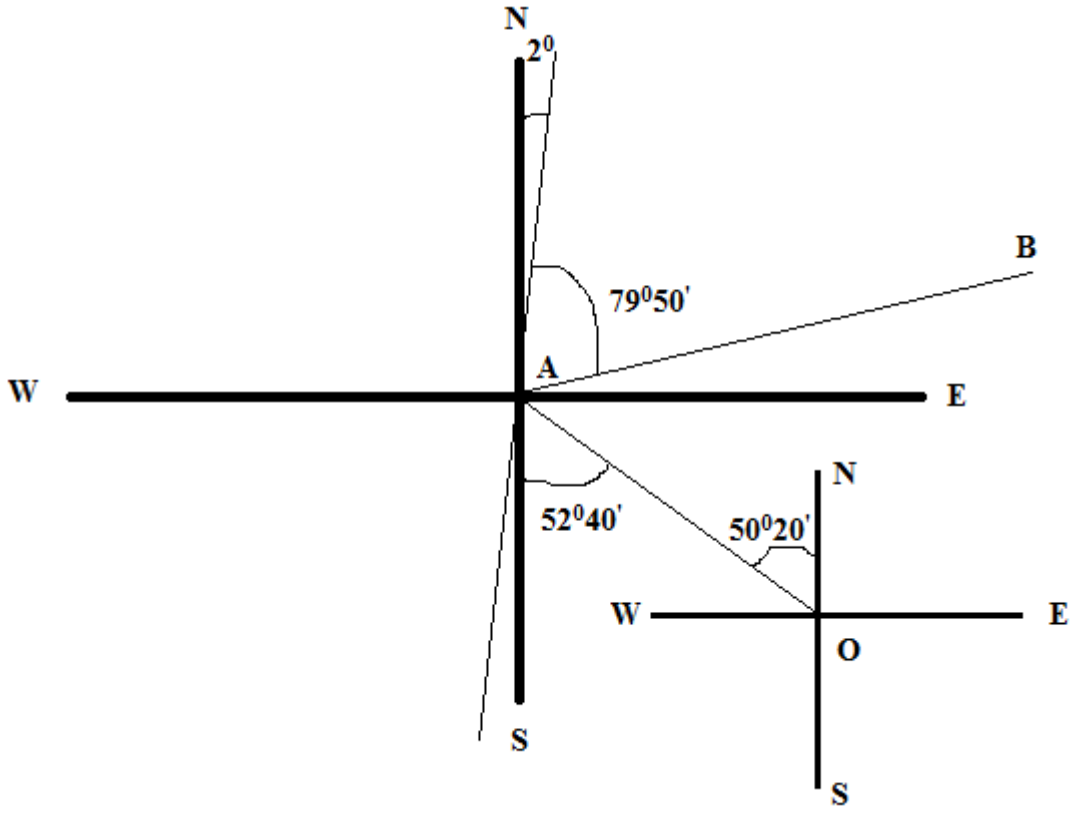
(B) N82°10′E

(C) N84°10′E

(D) N77°50′E

Ans (C) N84°10′E

- The given situation can be drawn as below:



At point O, there was no local attraction

Fore Bearing of OA = $360^0 - 50^0 20' = 309^0 40'$

So, Back bearing of OA = $309^0 40' - 180^0 = 129^0 40'$

Fore Bearing of AO = $180^0 - 52^0 40' = 127^0 20'$

» Error due to local attraction at A = $127^0 20' - 129^0 40' = -2^0 20'$

Hence, correction = $+ 2^0 20'$

Thus true FB of AB = $79^0 50' + 2^0 + 2^0 20' = 84^0 10'$ or **N84°10′E**

36. Consider the following complex function:

$f(z) = \frac{9}{(z-1)(z+2)^2}$

Which of the following is one of the residues of the above function ?

(A) –1

(B) 9/16

(C) 2

(D) 9

Ans (A) –1

The poles of the complex function can be calculated by putting the denominator = 0.

» z = 1 (pole of order 1) and -2 (pole of order 2) are the poles of the given function.

If f(z) has a simple pole at z = a, then

Residue of f(z) at z = a, Res f(a) = $\lim_{z \to a}[(z-a)f(z)]$

» Res f(1) = $\lim_{z \to 1}[(z-1)\frac{9}{(z-1)(z+2)^2}] = \lim_{z \to 1}[\frac{9}{(z+2)^2}] =$ **1**

If f(z) has a pole of order n at z = a, then

Residue of f(z) at z = a, Res f(a) = $\frac{1}{(n-1)!}\left\{\frac{d^{n-1}}{dz^{n-1}}\ [(z-a)^n\ f(z)]\right\}_{z=a}$

For z = -2 , n = 2 (pole order)

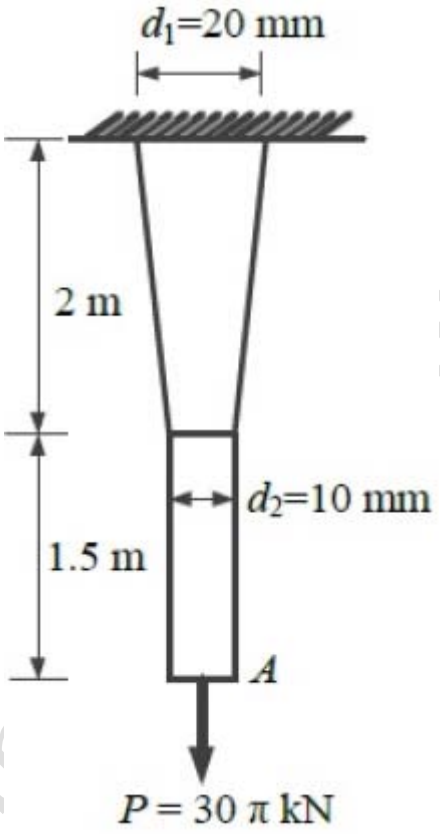
» Res f(-2) = $\frac{1}{(2-1)!}\left\{\frac{d^{2-1}}{dz^{2-1}}\ [(z+2)^2\ \frac{9}{(z-1)(z+2)^2}]\right\}_{z=-2}$

Res f(-2) = $\left\{\frac{d}{dz}\ [\frac{9}{(z-1)}]\right\}_{z=-2} = \left\{\frac{-9}{(z-1)^2}\right\}_{z=-2}$ = -9/9 = **-1**

So the residues of the given function are 1, -1.

Hence the option (A) is correct.

37. A tapered circular rod of diameter varying from 20 mm to 10 mm is connected to another uniform circular rod of diameter 10 mm as shown in the following figure. Both bars are made of same material with the modulus of elasticity, E = 2×10^5 MPa. When subjected to a load P= 30π kN, the deflection at point A is _____ mm.



Ans 15

- Let deflection in tapered circular rod = Δ_{tcr}

  Let deflection in circular rod = Δ_{cr}

  Deflection at point A = $\Delta = \Delta_{tcr} + \Delta_{cr}$

  $\Delta_{tcr} = [PL/(\pi d_1 d_2/4)E] = [30\ \pi \text{ x } 1000 \text{ x } 2000 / (\pi \text{ x } 10 \text{ x } 20/4)\ (2 \text{ x } 10^5)]$

  » Δ_{tcr} = 6 mm

  $\Delta_{cr} = [PL/(\pi d^2/4)E] = [30\ \pi \text{ x } 1000 \text{ x } 1500 / (\pi \text{ x } 10 \text{ x } 10/4)\ (2 \text{ x } 10^5)]$

  » Δ_{cr} = 9 mm

  Hence, $\Delta = \Delta_{tcr} + \Delta_{cr}$ = 15 mm

38. A 20m thick clay layer is sandwiched between a silty sand layer and a gravelly sand layer. The layer experiences 30 mm settlement in 2 years.

Given:

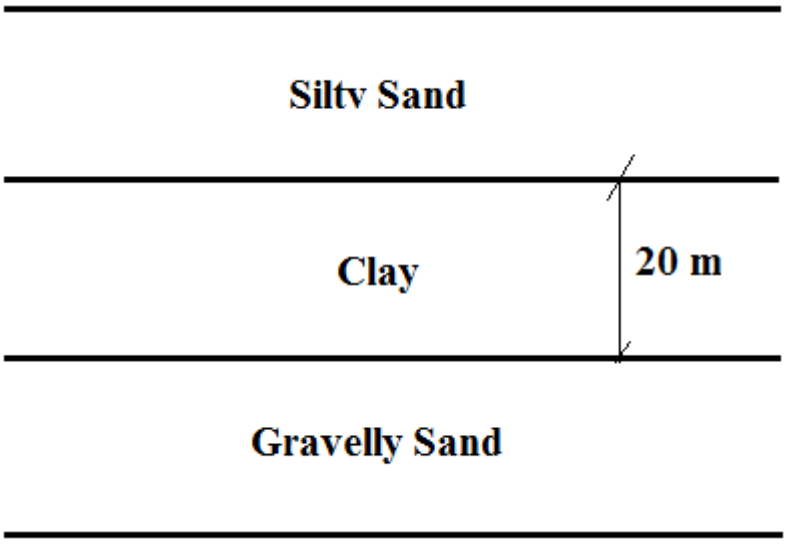
$$T_v = \begin{cases} \frac{\pi}{4}\left(\frac{U}{100}\right)^2 & \text{for } U \leq 60\% \\ 1.781 - 0.933\log_{10}(100 - U) & \text{for } U > 60\% \end{cases}$$

Where T_v is the time factor and U is the degree of consolidation in %.

If the coefficient of consolidation of the layer is 0.003 cm^2/s, the deposit will experience a total of 50 mm settlement in the next _____ years.

Ans 4.4 years

- The given details are shown in the figure below:



Length of drainage path, d = 20/2 = 10 m = 1000 cm

Now, $C_v = T_v\, d^2 / t$

» $t = (1000)^2\, T_v / 0.003$ (because $C_v = 0.003\ cm^2/s$)

$t = 3.33 \times 10^8\, T_v$ seconds ----------(A)

Now the layer settles for 30mm in 2 years; putting t = 2 years

$2 \times 365 \times 24 \times 60 \times 60 = 3.33 \times 10^8\, T_v$

» $T_v = 0.1894$

For 60% degree of consolidation (limiting value) i.e. U = 60%, time factor will be

$T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2 = 0.2826$

Hence for $T_v = 0.1894$, the degree of consolidation will be obtained by formula

$T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2$

» U = 49.12%

U = Given Settlement / Total settlement

» Total settlement = 30 / 0.4912 = 61.1 mm

Now, for settlement of 50 mm, degree of consolidation will be,

U = 50/61.1 = 81.83%

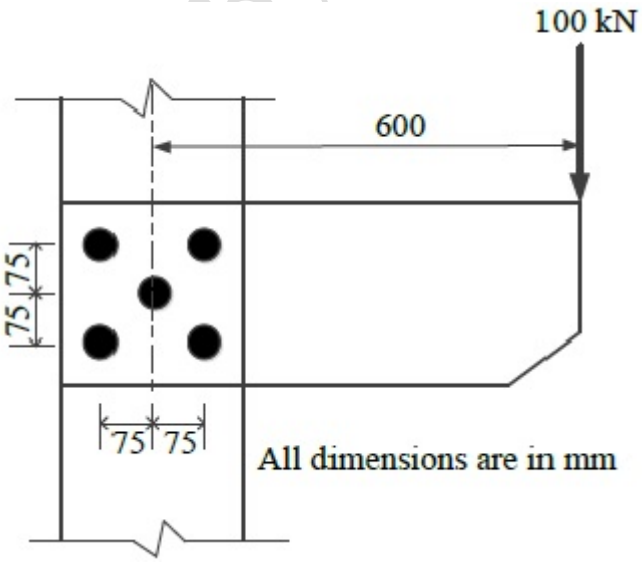
Now, for U > 60%

$T_v = 1.781 - 0.933\log_{10}(100 - U) = 0.6060$

» t = 3.33 x 10^8 x 0.6060 seconds = 6.4 years

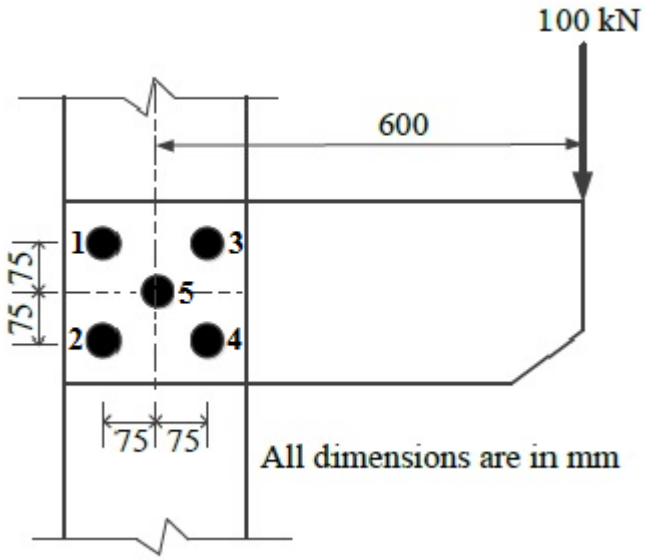
Hence, further time required for the 50 mm total settlement = 6.4 – 2 = **4.4 years**

39. A bracket plate connected to a column flange transmits a load of 100 kN as shown in the following figure. The maximum force for which the bolts should be designed is _____ kN.



Ans 156.20 kN

- The bolts are given numbers as shown below:



Direct shear on each bolt due to external load,

R_P = 100/5 = 20 kN (vertically downward)

Moment, M = Pe = 100 x 600 / 1000 = 60 kNm

Due to symmetry, the centre of gravity of the whole arrangement lies at the centre of the central bolt.

$\sum r^2 = \sum x^2 + \sum y^2 = 4 \text{ X } 0.075^2 + 4 \text{ X } 0.075^2 = 0.045 \text{ m}^2$

Load due to moment, $R_m = Mr/\sum r^2 = 1333.33 \text{ r kN}$

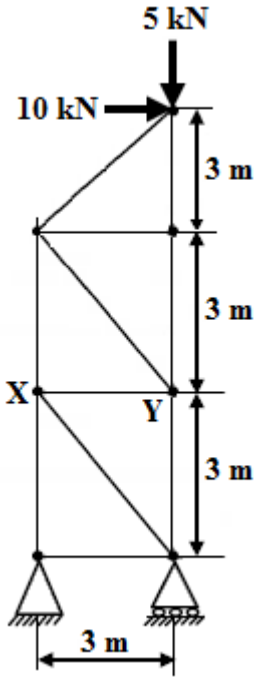
» Vertical component of $R_m = R_{mv} = Mx/\sum r^2 = 1333.33 \text{ x}$

Horizontal component of $R_m = R_{mH} = My/\sum r^2 = 1333.33 \text{ y}$

Bolt	x (m)	y (m)	R_{mv} (kN)	R_{mH} (kN)	H (kN)	$V = R_{mv} + R_P$ (kN)	$R = \sqrt{H^2 + V^2}$ (kN)
1	-0.075	0.075	-100	100	100	-80	128.06
2	-0.075	-0.075	-100	-100	-100	-80	128.06
3	0.075	0.075	100	100	100	120	156.20
4	0.075	-0.075	100	-100	-100	120	156.20
5	0	0	0	0	0	20	20

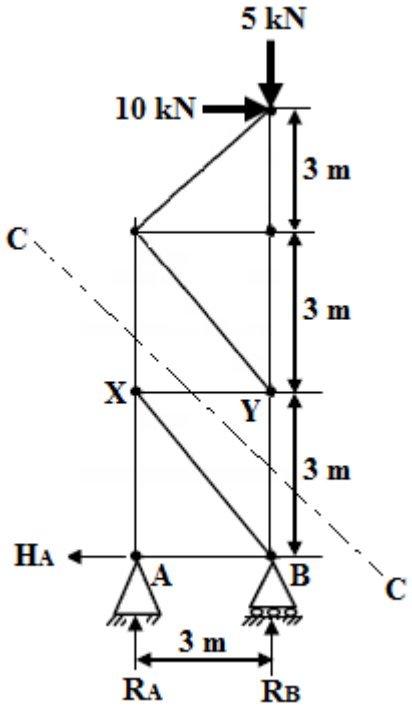
Hence maximum force for which the bolts should be designed is **156.20** kN.

40. For the 2D truss with the applied loads shown below, the strain energy in the member XY is _____ kN-m. For member XY, assume AE = 30kN, where A is cross-section area and E is the modulus of elasticity.



Ans 5 kNm

- The truss with the reactions is:



Clearly $H_A = 10$ kN

Taking moment about A,

$R_B\ (3) - 5\ (3) - 10\ (9) = 0$

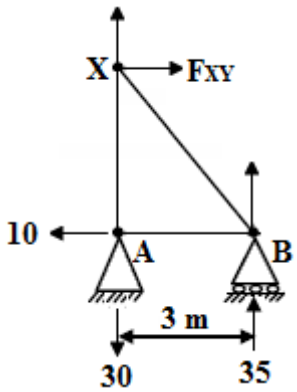
» $R_B = 35$ kN

$R_A + R_B = 5$

» $R_A = -30$ kN

Now applying method of intersection to determine force in member XY

Since only three independent equilibrium equations ($\sum F_x = 0, \sum F_y = 0, \sum M = 0$) can be applied to the isolated portion of the truss, select a section not passing through more than 3 members in which forces are unknown (section C-C).



$\sum H = 0$

» F_{XY} = 10 kN

Now, Strain energy/ volume = (1/2) (stress) (strain)

Strain energy = (1/2) (P/A) (P/AE) (AL) (because Volume = AL)

= $P^2L/2AE$ = 100 x 3 / (2 x 30) = **5 kNm** (because P = F_{XY} = 10)

41. The drag force, F_D, on sphere kept in a uniform flow field depends on the diameter of the sphere, D; flow velocity, V; fluid density, ρ; and dynamic viscosity,μ. Which of the following options represents the non-dimensional parameters which could be used to analyze this problem?

(A) F_D/VD and $\mu/\rho VD$

(B) $F_D/\rho VD^2$ and $\rho VD/\mu$

(C) $F_D/\rho V^2D^2$ and $\rho VD/\mu$

(D) $F_D/\rho V^3D^3$ and $\mu/\rho VD$

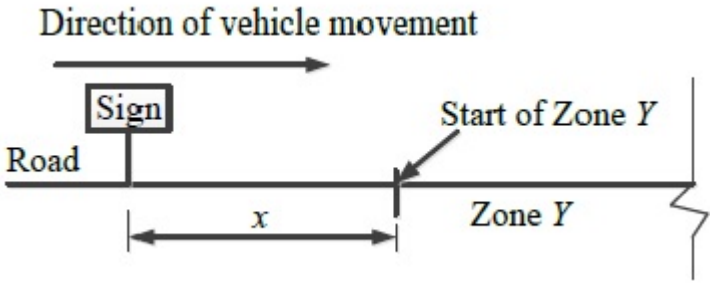
Ans (C) $F_D/\rho V^2D^2$ and $\rho VD/\mu$

- Clearly only option (C) has non-dimensional parameters

$F_D/\rho V^2D^2$ = kg m s^{-2} / (kg/m^3) $(ms^{-1})^2$ $(m)^2$ = Dimensionless

$\rho VD/\mu$ = (kg/m^3) (ms^{-1}) (m) / $kgm^{-1}s^{-1}$ = Dimensionless (units of dynamic viscosity,μ can be calculated using formula $\tau = \mu\, du/dy$)

42. A sign is required to be put up asking drivers to slow down to 30 km/h before entering Zone Y (see figure). On this road, vehicles require 174 m to slow down to 30 km/h (the distance of 174 m includes the distance travelled during the perception – reaction time of drivers). The sign can be read by 6/6 vision drivers from a distance of 48 m. The sign is placed at a distance of 'x' m from the start of Zone Y so that even a 6/9 vision driver can slow down to 30 km/h before entering the zone. The minimum value of 'x' is _____ m.



Ans 142

- Distance required by vehicles to slow down = 174 = x + reading distance

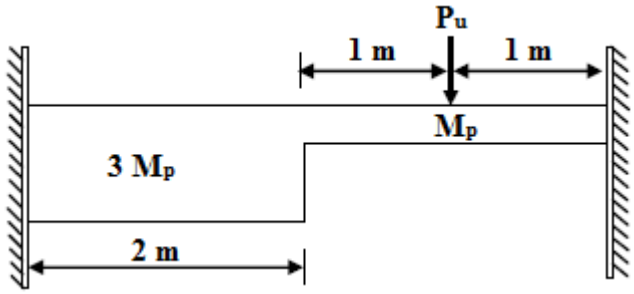
  For 6/6 vision, Reading distance = 48 m

  6/9 vision means that the letters in the chart or sign board which should be read at 9 metres for 6/6 vision will be readable at 6 metres i.e. at closer distance.

  Thus, reading distance for 6/9 vision = 48 X 6/9 = 32 m

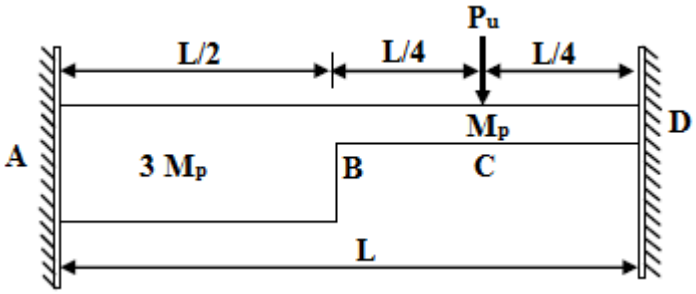
  Hence, x = 174 – 32 = **142 m**

43. For formation of collapse mechanism in the following figure, the minimum value of P_u is cM_p/L. M_p and $3M_p$ denote the plastic moment capacities of beam sections as shown in this figure. The value of 'c' is_____.

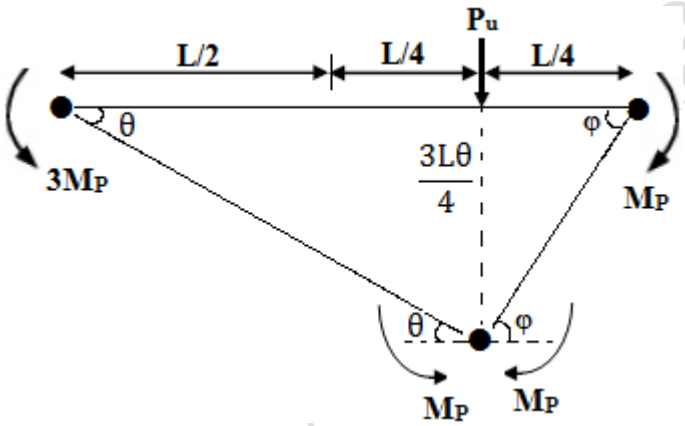


Ans 13.33

- Let the total length of the beam = L



There are two possible collapse mechanisms. In the first mechanism, hinges may form at A, C and D.



From the figure above,

$3L\theta/4 = L\varphi/4$

» $\varphi = 3\theta$

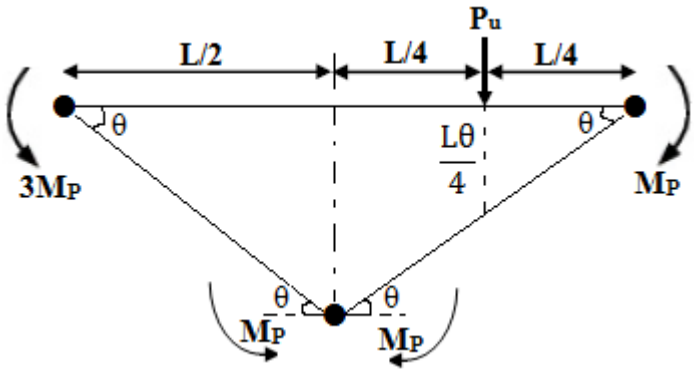
Work done by the load = P_u $(3L\theta/4)$

Work absorbed by the hinges = $M_P\ \varphi + M_P\ (\theta + \varphi) + 3M_P\theta = 10M_P\theta$

For equilibrium, $P_u\ (3L\theta/4) = 10M_P\theta$

» $P_u = 40M_P/3L$ ----------(A)

In the second mechanism, hinges may form at A, B and D. The hinge at B will work corresponding to the least moment of resistance at B, i.e. M_P.



For equilibrium, $P_u\ (L\theta/4) = 3M_P\theta + M_P\theta + M_P\theta + M_P\theta$

» $P_u = 24M_P/L$ ----------(B)

The mechanism is one which gives minimum of the collapse load (From (A) & (B)).

Thus $P_u = 40M_P/3L = cM_P/L$

Hence, c = 40/3 = **13.33**

44. In a survey work, three independent angles X,Y and Z were observed with weights W_X, W_Y, W_Z respectively. The weight of the sum of angles X,Y and Z is given by:

(A) $1 / \left(\frac{1}{W_X} + \frac{1}{W_Y} + \frac{1}{W_Z}\right)$

(B) $\left(\frac{1}{W_X} + \frac{1}{W_Y} + \frac{1}{W_Z}\right)$

(C) $W_X + W_Y + W_Z$

(D) $W_X^2 + W_Y^2 + W_Z^2$

Ans (A) $1 / \left(\frac{1}{W_X} + \frac{1}{W_Y} + \frac{1}{W_Z}\right)$

- Weight of i^{th} observed quantity = $W_i = 1/\sigma_i^2$ (where σ_i^2 = variance of i^{th} observation and σ_i = standard deviation of i^{th} observation)

» $W_X = 1/\sigma_x^2$, $W_Y = 1/\sigma_y^2$, $W_Z = 1/\sigma_z^2$

Total variance of sum of angles = $\sigma^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$

$1/W = \frac{1}{W_X} + \frac{1}{W_Y} + \frac{1}{W_Z}$ (where W = weight of sum of angles)

» **$W = 1 / \left(\frac{1}{W_X} + \frac{1}{W_Y} + \frac{1}{W_Z}\right)$**

45. The acceleration – time relationship for a vehicle subjected to non-uniform acceleration is,

$$\frac{dv}{dt} = (\alpha - \beta v_0)e^{-\beta t}$$

where, v is the speed in m/s, t is the time in s, α and β are parameters, and v_0 is the initial speed in m/s. If the accelerating behavior of a vehicle, whose driver intends to overtake a slow moving vehicle ahead, is descried as,

$$\frac{dv}{dt} = (\alpha - \beta v)$$

Considering $\alpha = 2$ m/s^2, $\beta = 0.05$ s^{-1} and $\frac{dv}{dt} = 1.3$ m/s^2 at t = 3s, the distance (in m) travelled by the vehicle in 35 s is _____.

Ans 900.96 m

- $\frac{dv}{dt} = (\alpha - \beta v_0)e^{-\beta t}$

As $\frac{dv}{dt} = 1.3$ m/s^2 at t = 3s and $\alpha = 2$ m/s^2, $\beta = 0.05$ s^{-1}

» $1.3 = (\alpha - \beta v_0)e^{-3\beta}$

$\alpha - \beta v_0 = 1.51$ & $v_0 = 9.8$

Further $\int dv = \int (\alpha - \beta v_0)e^{-\beta t}dt$

$\int dv = \int 1.51\, e^{-0.05t}dt$

$v = \frac{1.51e^{-0.05t}}{-0.05} + C$ (where C – Arbitrary constant)

$v = -30.2\, e^{-0.05t} + C$ ----------(A)

When t = 0, $v = v_0 = 9.8$ m/s^2; Putting these values in equation (A), we get

$C = 9.8 + 30.2 = 40$

» $v = 40 - 30.2\, e^{-0.05t}$

$\frac{dx}{dt} = 40 - 30.2\, e^{-0.05t}$

» $\int dx = \int (40 - 30.2\, e^{-0.05t})dt$

$x = 40t + 604\, e^{-0.05t} + D$ ----------(B)

At t = 0, x = 0; Putting these values in equation (B), we get

D = - 604

» $x = 40t + 604\, e^{-0.05t} - 604$

Hence, when t = 35 s

$x = 40 \times 35 + 604\,(e^{-0.05 \times 35} - 1) = 1400 - 499.04 =$ **900.96 m**

46. Consider a primary sedimentation tank (PST) in a water treatment plant with Surface Overflow Rate (SOR) of 40 $m^3/m^2/d$. The diameter of the spherical particle which will have 90 percent theoretical removal efficiency in this tank is _____ μm. Assume that settling velocity of the particles in water is described by stokes's Law.

Given: Density of water = 1000 kg/m^3; Density of particle = 2650 kg/m^3;

g = 9.81 m/s^2; Kinematic viscosity of water (ν) = 1.10×10^{-6} m^2/s

Ans 22.58 μm

- The percentage of particles removed, P, with a settling velocity of v_s in a sedimentation tank designed with an overflow rate of v_0 is

$P = (v_s / v_0) \times 100\%$

Here, $v_0 = 40\ m^3/m^2/d = 40/(24 \times 60 \times 60) = 4.63 \times 10^{-4}$ m/s

» $v_s = 0.9 \times 4.63 \times 10^{-4} = 4.17 \times 10^{-4}$ m/s

Now, from stoke's law, settling velocity of spherical particle is given by,

$$v_s = \frac{g\,(\rho_s - \rho_l)d^2}{18\,\mu}$$

Here, ρ_s = 2650 kg/m^3; ρ_l = 1000 kg/m^3; d = dia of spherical particle

μ = dynamic viscosity = $\rho\,\upsilon = 1.10 \times 10^{-6} \times 10^3 = 1.10 \times 10^{-3}$ m^2/s

$$\text{» } 4.17 \times 10^{-4} = \frac{(9.81)\,(2650 - 1000)d^2}{18 \times 1.10 \times 10^{-3}}$$

$d = 2.258 \times 10^{-5}$ m = 22.58 μm

47. On a circular curve, the rate of superelevation is e. While negotiating the curve a vehicle comes to a stop. It is seen that the stopped vehicle does not slide inwards (in the radial direction). The coefficient of side friction is f. Which of the following is true:

(A) $e \le f$

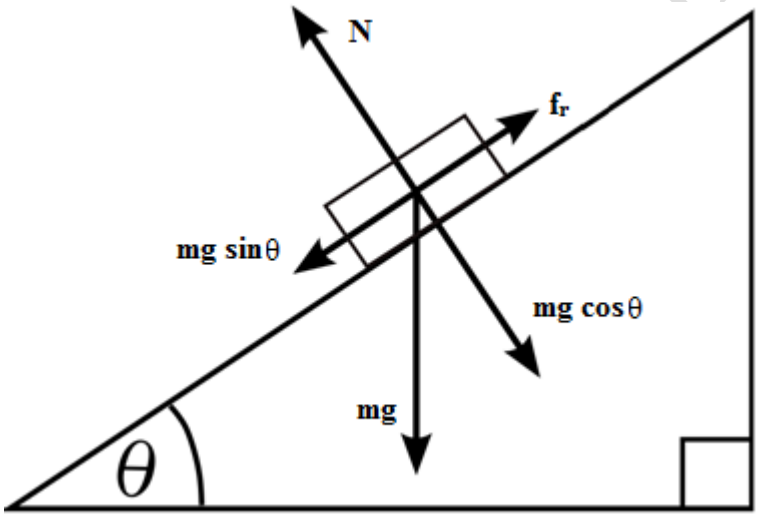
(B) $f < e < 2f$

(C) $e \ge 2f$

(D) none of the above

Ans (A) $e \le f$

- The free body diagram of the vehicle is shown below:



As the vehicle does not slide inwards,

Frictional force ≥ Inward Force

» $f_r \ge mg \sin\theta$

But $f_r = f\,N$ and $N = mg\cos\theta$

» $f\,mg\cos\theta \ge mg\sin\theta$

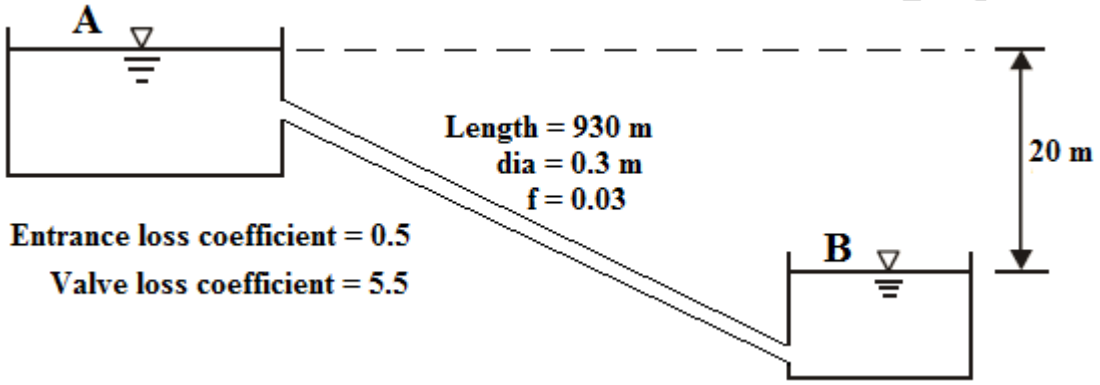
$f \ge \tan\theta$

» **$f \ge e$** (because Rate of superelevation, $e = \tan\theta$)

48. Two reservoirs are connected through a 930 m long, 0.3 m diameter pipe, which has a gate valve. The pipe entrance is sharp (loss coefficient= 0.5) and the valve is half-open (loss coefficient = 5.5). The head difference between the two reservoirs is 20 m. Assume the friction factor for the pipe as 0.03 and g =10 m/s^2. The discharge in the pipe accounting for all minor and major losses is_____ m^3/s.

Ans 0.1413

- The given conditions are as shown below:



Applying Bernoulli's Equation at A and B,

Energy at A – Energy Lost = Energy at B

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A - \text{Energy Lost} = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B \text{ ----------(A)}$$

Here, $P_A = P_B$; $V_A = V_B = 0$ (negligible); $Z_A - Z_B = 20$

Energy Lost = Head loss due to (friction + Entrance Loss + Valve half open loss + Exit Loss)

Lost head = $KV^2/2g$ (where V is the fluid velocity in pipe)

$$\text{Head Loss due to friction} = \frac{fLV^2}{2gd} = \frac{0.03 \times 930 \times V^2}{2 \times 10 \times 0.3}$$

$$\text{Entrance Loss} = \frac{0.5 \times V^2}{2 \times 10} \text{ ; Valve half open loss} = \frac{5.5 \times V^2}{2 \times 10} \text{ ;}$$

$$\text{For exit loss, K = 1, so Exit Loss} = \frac{V^2}{2 \times 10}$$

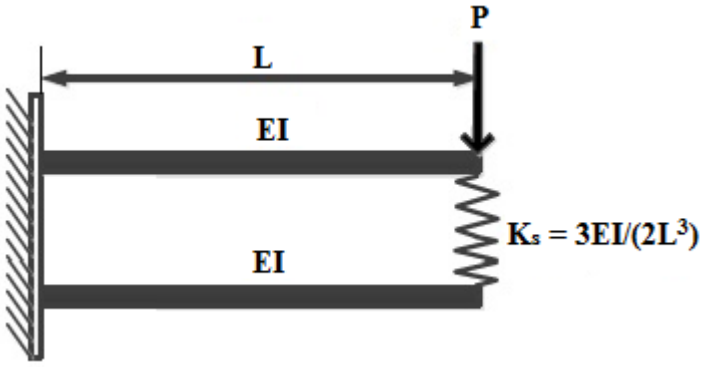
Putting all values in equation (A), we get

$$20 = \frac{V^2}{2x\ 10} + \frac{0.03\ x\ 930\ x\ V^2}{2x\ 10\ x\ 0.3} + \frac{0.5\ x\ V^2}{2\ x\ 10} + \frac{5.5\ x\ V^2}{2\ x\ 10}$$

» V = 2 m/s

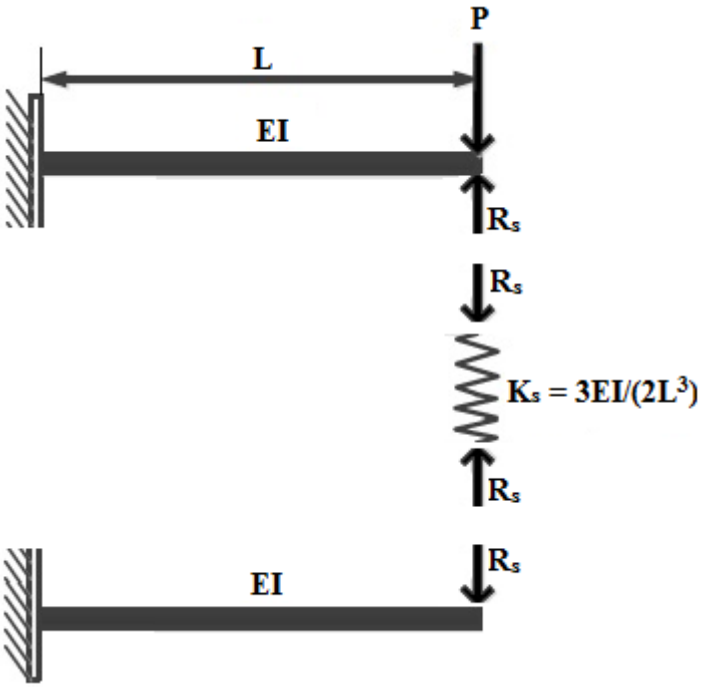
Hence, discharge = AV = $(\pi d^2/4)$ V = $[\pi\ (.3)^2/4]$ x 2 = **0.1413 m/s**

49. Two beams are connected by a linear spring as shown in the following figure. For a load P as shown in the figure, the percentage of the applied load P carried by the spring is _____.

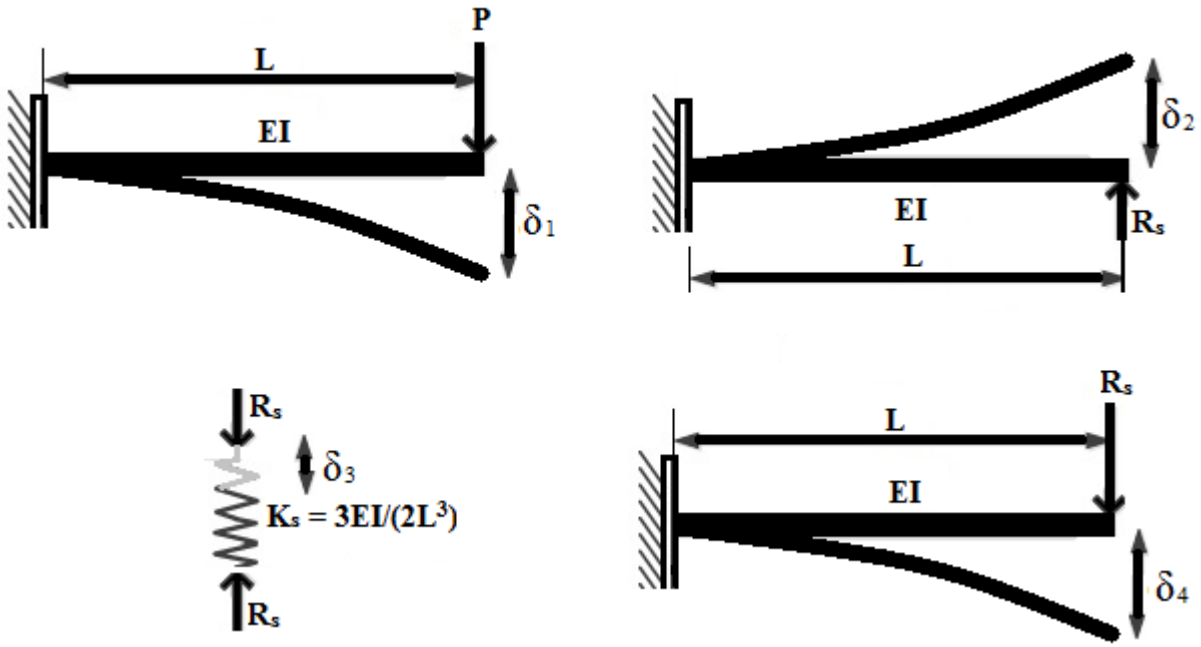


Ans 25%

- The free body diagram is shown below: R_s is the reaction in spring.



The total deflections due to load are shown in the diagram given below:



δ_1 = Downward deflection due to load P (causing compression in spring) = $PL^3/3EI$

δ_2 = Upward deflection due to reaction R_s (causing elongation in spring) = $R_sL^3/3EI$

δ_3 = Shortening in spring due to reaction R_s = R_s/K_s = $2R_sL^3/3EI$

δ_4 = Downward deflection due to load R_s (causing elongation in spring) = $R_sL^3/3EI$

From Compatibility Equation,

$\delta_3 = \delta_1 - \delta_2 - \delta_4$

$2R_sL^3/3EI = PL^3/3EI - R_sL^3/3EI - R_sL^3/3EI$

» R_s = P/4 = 25% of P

50. The smallest and largest Eigen values of the following matrix are:

$$\begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix}$$

(A) 1.5 and 2.5

(B) 0.5 and 2.5

(C) 1.0 and 3.0

(D) 1.0 and 2.0

Ans (D) 1.0 and 2.0

- For Eigen values, $| A - \lambda I | = 0$

$$\begin{vmatrix} 3-\lambda & -2 & 2 \\ 4 & -4-\lambda & 6 \\ 2 & -3 & 5-\lambda \end{vmatrix} = 0$$

$(3 - \lambda) [(-4 - \lambda) (5 - \lambda) - 6 (-3)] + 2 [4 (5 - \lambda) - 6 (2)] + 2 [4 (-3) - 2 (-4 - \lambda)] = 0$

» $\lambda^3 - 2\lambda^2 + 5\lambda - 2 = 0$

Using trial and error method, $\lambda = 1$ satisfies the equation.

» $(\lambda - 1) (\lambda^2 - 3\lambda + 2) = 0$

$(\lambda - 1) (\lambda - 1) (\lambda - 2) = 0$

So, $\lambda = 1, 2, 2$

Hence, smallest eigen value = 1 and largest eigen value = 2.

51. A hydraulic jump is formed in a 2m wide rectangular channel which is horizontal and frictionless. The post-jump depth and velocity are 0.8 m and 1m/s, respectively. The pre-jump velocity is _____ m/s. (use g = 10 m/s^2)

Ans 4.82

- $y_1 = \frac{y_2}{2} (- 1 + \sqrt{1 + 8F_2^2})$

Here, $y_2 = 0.8$ m, $V_2 = 1$ m/s; $F_2 = V_2/\sqrt{gy_2} = 1/\sqrt{10 \times 0.8} = 0.3535$

» $y_1 = \frac{0.8}{2} (- 1 + \sqrt{1 + 8(0.3535)^2}) = 0.166$ m

Now, Discharge = AV

» $A_1 V_1 = A_2 V_2$

$2 \times 0.166 \times V_1 = 2 \times 0.8 \times 1$

» $V_1 = 4.82$ m/s

52. The quadratic equation $x^2 - 4x + 4=0$ is to be solved numerically, starting with the initial guess $x_0 = 3$. The Newton - Raphson method is applied once to get a new estimate and then the Secant method is applied once using the initial guess and this new estimate. The estimated value of the root after the application of the secant method is _____.

Ans 2.33

- According to Newton – Raphson method for f(X) = 0,

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$

$$\text{» } X_1 = X_0 - \frac{f(X_0)}{f'(X_0)}$$

Here, $f(X) = x^2 - 4x + 4$

» $f'(X) = 2x - 4$

Now, $X_0 = 3$, $f(X_0) = f(3) = 1$ and $f'(X_0) = f'(3) = 2$

$$X_1 = 3 - \frac{1}{2} = \frac{5}{2}$$

According to Secant method for f(X) = 0,

$$X_{n+1} = X_n - \frac{f(X_n)(X_n - X_{n-1})}{f(X_n) - f(X_{n-1})}$$

$$X_2 = X_1 - \frac{f(X_1)(X_1 - X_0)}{f(X_1) - f(X_0)}$$

$$f(X_1) = (5/2)^2 - 4(5/2) + 4 = \frac{1}{4}$$

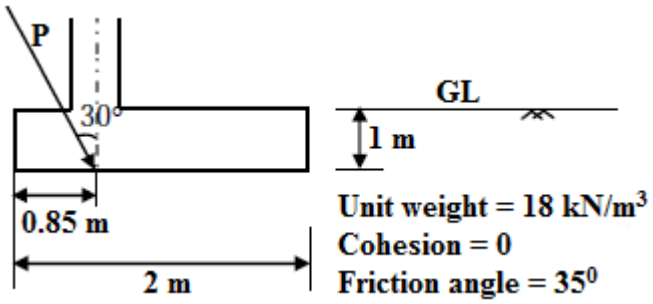
$$\text{» } \mathbf{X_2 = \frac{5}{2} - \frac{\left(\frac{1}{4}\right)\left(\left(\frac{5}{2}\right) - 3\right)}{\frac{1}{4} - 1} = 2.33}$$

53. A square footing (2 m × 2 m) is subjected to an inclined point load, P as shown in the figure below. The Water table is located well below the base of the footing. Considering one-way eccentricity, the net safe load carrying capacity of the footing for a factor of safety of 3.0 is _____ kN.

The following factors may be used:

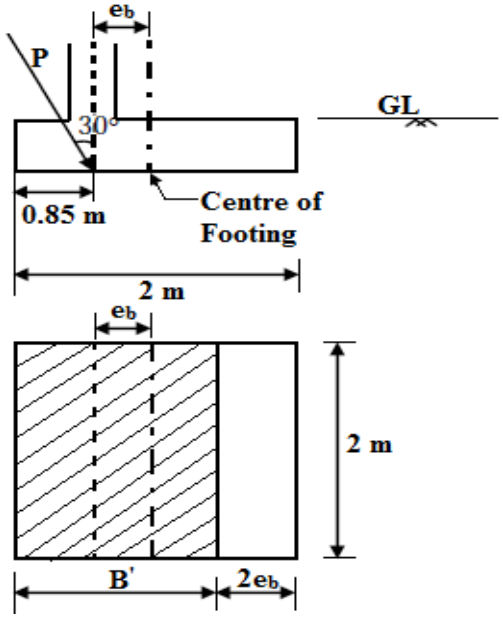
Bearing capacity factors: $N_q = 33.3$, $N_\gamma = 37.16$; Shape factors : $F_{qs} = F_{\gamma s} = 1.314$;

Depth factors: $F_{qd} = F_{\gamma d} = 1.113$; Inclination factors: $F_{qi} = 0.444$, $F_{\gamma i} = 0.02$



Ans 439.56 kN

- The effective area due to eccentric loading is shown below:



e_b = 1 – 0.85 = 0.15 m

» $B^{'}$ = 2 – 2 x 0.15 = 1.70 m

Using Meyerhof's method for eccentrically loaded foundations, we get

Ultimate Bearing Capacity, $q_u = cN_c\ F_{cs}\ F_{cd}\ F_{ci} + qN_q\ F_{qs}\ F_{qd}\ F_{qi} + 0.5B^{'}\gamma\ N_\gamma\ F_{\gamma s}\ F_{\gamma d}\ F_{\gamma i}$

q_u = [18 x 1 x 33.3 x 1.314 x 1.113 x 0.444 + 0.5 x 1.7 x 18 x 37.16 x 1.314 x 1.113 x 0.02] (where $q = \gamma D = 18 \times 1 = 18\ kN/m^2$)

» $q_u = 405.845\ kN/m^2$

Therefore, Net ultimate Bearing Capacity = $q_{nu} = q_u - \gamma D = 387.845\ kN/m^2$

» Net Safe Bearing Capacity, $q_{ns} = q_{nu} / 3 = 129.282\ kN/m^2$

Hence net safe load carrying capacity of footing = q_{ns} x $B^{'}$ x 2 = **439.56 kN**

54. The directional derivative of the field $u(x,y,z) = x^2 - 3yz$ in the direction of the vector $(i + j - 2k)$ at point $(2,-1,4)$ is _____.

Ans -5.715

- Here, $u = x^2 - 3yz$

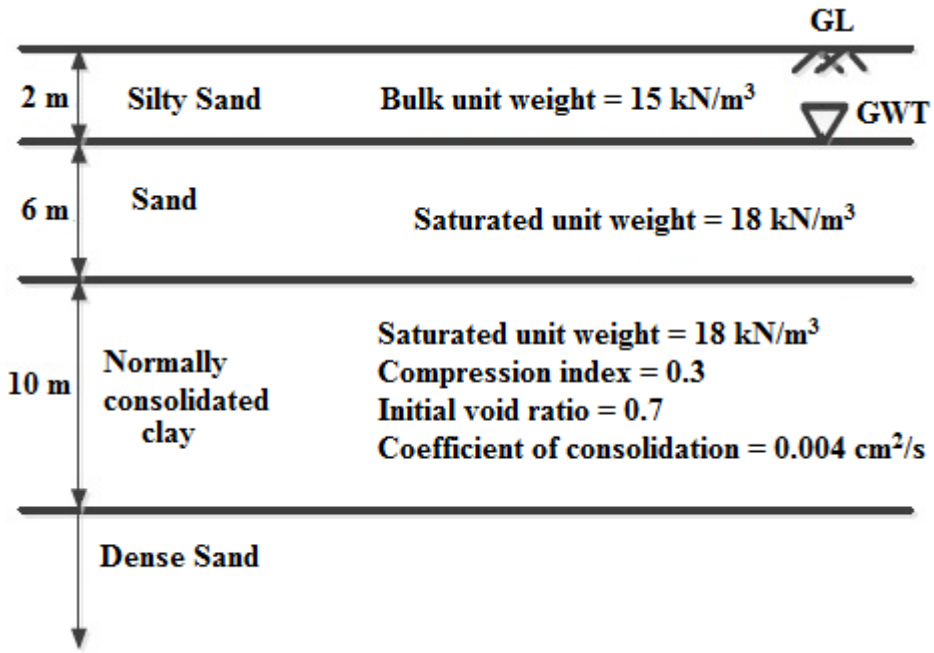
  Gradient $u = \nabla u = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) u = 2xi - 3zj - 3yk$

  At point P(2,–1,4), $\nabla u = 4i - 12j + 3k$

  Directional derivative of 'u' in the direction of vector 'a' is given by

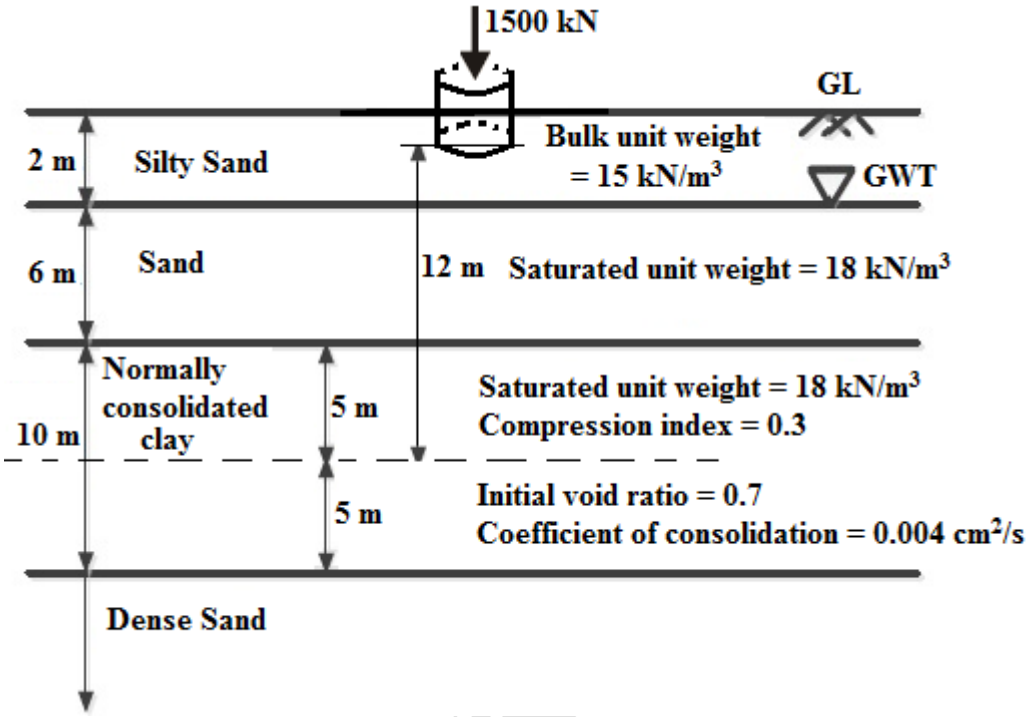
  $D = \frac{1}{|a|}\, a.\nabla u = \frac{1}{\sqrt{1^2+1^2+2^2}}(i + j - 2k).(4i - 12j + 3k) =$ **-5.715**

55. A water tank is to be constructed on the soil deposit shown in the figure below. A circular footing of diameter 3 m and depth of embedment 1m has been designed to support the tank. The total vertical load to be taken by the footing is 1500 kN. Assume the unit weight of water as 10 kN/m^3 and the load dispersion pattern as 2V:1H. The expected settlement of the tank due to primary consolidation of the clay layer is _____mm.



Ans 53.24 mm

- The given conditions are shown in the diagram below:



Initial pressure at the centre of clay layer

$\bar{\sigma}_0$ = 2 x 15 + 6 x (18-10) + 5 x (18-10) (because of GWT, $\gamma_{sub} = \gamma - \gamma_w$ is taken)

» $\bar{\sigma}_0$ = 118 kN/m^2

Pressure at the base of footing, q = 1500/(π x 3^2/4) kN/m^2

Pressure increase at the centre of clay layer because of 2V:1H load distribution is given by, $\Delta\bar{\sigma} = \frac{q\,D^2}{(D+Z)^2}$

» $\Delta\bar{\sigma} = \frac{1500 \text{ x } 3^2}{\left(\frac{\pi}{4}\right) \text{ x } 3^2 \text{x } (3+12)^2}$ = 8.488 kN/m^2

Thus, Settlement (S) = $\frac{C_c H_0}{1+e_0} \log\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right) = \frac{0.3 \text{ x } 10}{1+0.7} \log\left(\frac{118 + 8.488}{118}\right)$

» S = 0.05324 m = 53.24 mm